The Approximation Of Nonlinear Function using Daubechies and Symlets Wavelets

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Abstract: The phenomena and real world problems usually can be formulated as a representation of the problem of function approximation, which is to estimate the value of a function f(*x*), based on the relationship or pattern of the input-output data, that is sequences of $y_i = f(x_i)$. In practice, some applications related to the approximation of functions such as the problems of pattern classification, regression analysis, reconstruction signals, and identification systems. The purpose of this research is to compare the performance of Daubechies and Symlets wavelet types to estimate nonlinear functions. The characteristics of the Daubechies and Symlets wavelet functions are smooth, regular, have a compact of the support and lengthy of the filters, and an explicit the formula so it's good to handle smooth curves, reconstruct of signals, longer filtering processes, easy and fast on computing process. The advantages of the Daubechies and Symlets wavelet characteristics will be used as the basis for approximating non-linear functions. Numeracally, based on the means square error (MSE) indicator, the results of this research provide an overview of the accuracy of wavelet-based approximation by Daubechies and Symlets wavelets type for approximation of the nonlinear function which is approximated is very significant.

1 INTRODUCTION

Some of the phenomena or real problems can be formulated as a representation of the problem of function approximation. In general, sample data from an observation or a result study usually in the form of input-output ordered pairs set. Zainuddin and Pauline (2011) stated that the main problem related to the function approximation is to estimate the value of a function based on the relationship that exists in the input-output data set represented by the pattern. Obviously, the function approximation can be interpreted as an attempt to estimate a function value based on a relationship or pattern that is formed in the representation of the relationship between the input-output values in the sample data. In general, some practical applications with regard to function approximations are problems with pattern classification, data mining, signal reconstruction, and identification systems.

Wavelet analysis and wavelet transformation are branches of mathematical studies that have been applied to various fields of science. Early in its development, wavelets were initiated as a combination of pure mathematical ideas (harmonic analysis, functional analysis, approximation theory, fractal geometry) and applied mathematics (signal processing, and mathematical physics). Various studies have been carried out related to this topic. Some publications related to wavelet applications and wavelet transformation include forecasting and prediction problems (Matsumoto et al, 2007), problems of filtering data (Ahamada et al, 2010), adaptive data and singularity problems (Bruzda, 2004), trend analysis issues(Alexsandrov et al, 2008), stationary and non-stationary data problems (Lineesh, 2010), inflation and price index issues (Ysusi, 2009), multiresolution issues (Alves et al 2002), growth problems and cycles of agricultural products (Chen, 2002), and variance and data correlation issues (Gallegati et al, 2005).

The wavelet function consists of several types such as Haar, Daubechies, Morlet, Mexican, and B-spline wavelets. Each type of wavelet has advantages and disadvantages, especially related to the form of a function as a wavelet representation, the form of wavelet curves, support areas, and so on. At the application level, there is a certain type of wavelet which because of its superiority and some properties it has become the reason researchers use it as a tool or base of analysis in the object of research such as Haar wavelet, Daubechies, Symlet, or wavelet B-spline.

Liu and Din (2016) revealed that Daubechies wavelets have the advantage of having orthogonality and compact support. These two properties can numerically improve the analysis performance in terms of accuracy and ensure stability in the computation process, and the existence of the wavelet scaling function on [0, + ∞] used as the basis functions for approximations. On the other hand, Symlets constitute a family of almost symmetric wavelets proposed by Daubechies by modifying the construction of the dbN. Therefore apart from the symmetry, the other properties of the two families are similar. The fundamental difference is only in the nature where the Daubechies wavelet is asymmetrical and the Symlet wavelet is almost symmetric, with the higher the order the higher the level of symmetry. On the application side, wavelet Symlet has vanishing moments at most if given wide support to be one of the reasons researchers using the wavelet Symlets as a tool in its research analysis.

Yadav and Mehra (2016) revealed that the superiority of Daubechies and Symlet wavelets can improve accuracy based on the MSE indicator in the denoising ECG signal process. Comprehensively, the advantages of the properties of these two family wavelets are summarized by Misiti et al. (2007: 92) such as regular arbitrary, orthogonal with compact support, the arbitrary numbers of zero moments, the existence of scaling function, orthogonal analysis, exact reconstruction, continous and discrete transformation formulas, and fast algorithm.

Therefore, based on the data characteristic of observation/ research which is generally non-linear, the extent of the application of wavelet/wavelet transformation in various application fields, as well as the superiority of wavelet characteristics of Daubechies and Symlets, then wavelet Daubechies and Symlets are selected and used as the basis for approximating nonlinear functions..

2 WAVELET APPROXIMATION

2.1 Representation of The Approximation Function

Many phenomena from various application domains are representations of the approximation of functions problems. The results of observations from a research or real problem usually can be represented as a set of input-output ordered pairs. In this respect, the problem of approximating the function is to estimate or estimate the value based on the pattern of the relationship between the input-ouput present in the sample data.

Furthermore, in general, the representation of real phenomena can be formulated using a continuous function. To simplify the problems, let us assume that the function space is assumed to be finite. In this case, one representative function space is Hilbert space. Let the function be

$$f(x) = \sum_{i=1}^{N} w_i h_i(x) \qquad (1)$$

where w_i indicates weight parameter to be updated and $h_i(x)$ indicates the basis of the selected function. Some candidate of base functions such as polynomial, trigonometric, exponential, and orthogonal functions. One of the functional types that has recently been used as a base function is the wavelet function (Zainuddin and Pauline, 2011).

In Hilbert space, the approximation using the wavelet function is defined as follows

$$f(x) = \sum_{i=1}^{N} w_i \psi(a_i(x - b_i)) \qquad (2)$$

with a_i and b_i respectively states the dilated and translational coefficients of the function of the mother wavelet ψ.

2.2 Wavelet and Wavelets Transformation

Wavelets are a class of functions used to localize a given function in two ways, ie. position (time) and scale (frequency). This ability that causes wavelets to have advantages over Fourier transforms that are widely applied in data processing such as signal processing and time series analysis (Bahri, 2016a, b).

Wavelet is defined as a shortwave that concentrates its energy in space and time or a limited or localized wave (Figure 1). Unlike the wave which is a function of periodic space and time.

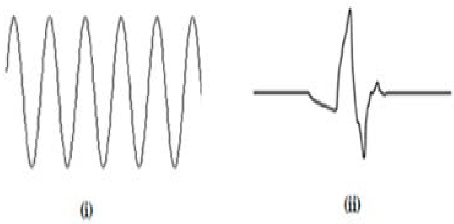

Figure 1: Graph of (i) *wave* and (ii) *wavelet*

Mathematically, the wavelet is a family of functions constructed from the translation and dilation of a function, defined as follows (Debnath, 2002; Daubechies, 1992):

$$\psi_{a,b}(t)=|a|^{-\frac{1}{2}}\psi\left(\frac{t-b}{a}\right), \quad a,b\in\mathbb{R} \text{ dan } a\neq 0, \qquad (3)$$

with a mother wavelet ψ, a represents the scaling (dilation) parameter that determines the degree of compression or scale, and b represents the translation parameter that determines the time location of the wavelet. The function ψ on (3) is called the mother wavelet, if it verifies the following admissibility conditions:

$$\text{(i)} \int_{-\infty}^{\infty} \psi(t)\, dt=0 \qquad (2.4)$$

$$\text{(ii)} \int_{-\infty}^{\infty} \psi^2(t)\, dt=1 \qquad (2.5)$$

$$\text{(iii)}\ C_\psi=\int_{-\infty}^{\infty} \frac{|\psi-w|^2}{w}\, dw<\infty \qquad (2.6)$$

2.1.1 The Types of Wavelet Transform

Wavelet transformation can be distinguished into two types: continuous wavelet transformation and discrete wavelet transformation (Boggess and Narcowich, 2001: 184; Mohlenkamp, 2008: 31; Daubechies 1992: 7-8 and Burrus, 1998: 7-9).

If the scaling parameter a and the translation parameter b are continuous variables on the $\mathbb{R}$ field, with $a\neq 0$ on (3), we defined continuous wavelet transform type which is given by the following equation:

$$\left[T^{wav} f\right](a,b)=|a|^{-\frac{1}{2}}\int_{-\infty}^{\infty} f(t)\ \psi\left(\frac{t-b}{a}\right) dt\ . \qquad (4)$$

A function f can be reconstructed from the wavelet transformation equation using the "identity resolution" equation or the following wavelet transformation inverse:

$$f = C_\psi^{-1}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{a^2}\left\langle f,\psi_{a,b}\right\rangle \psi_{a,b}\ da\ db$$

where $\psi_{a,b}(t) = |a|^{-1/2}\psi\left(\frac{t-b}{a}\right)$ and $\langle\cdot,\cdot\rangle$ indicates the operation of the inner product on $L^2(\mathbb{R})$. Whereas the constant C_ψ depends only on the variable ψ and is given by the following equation:

$$C_\psi=2\pi\int_{-\infty}^{\infty}\left|\hat{\psi}(\xi)\right|^2 |\xi|^{-1} d\xi \qquad (5)$$

with $\hat{\psi}(\xi)$ denoting the Fourier transform of wavelet $\psi(t)$ and in order that Equation (5) is undefined it is assumed that $C_\psi<\infty$. If $\psi(t)$ is at $L^1(\mathbb{R})$ then the function $\psi(t)$ is continuous so $\hat{\psi}(\xi)$ that will be finite only if $\hat{\psi}(0)=0$, that is

$$\int_{-\infty}^{\infty}\psi(x)\,dx=0$$

If the scaling parameter a and the translation parameter b are discrete numbers on (3), we have the discrete wavelet transform type. Since a is selected to be discrete (positive or negative) it can be selected a as the exponential of the dilated parameter $a_0>1$, that is $a=a_0^m$. The difference in the values of m corresponds to the width of the wavelet time area. This shows that the discretization of the translation parameter value b depends on the number m. Short wavelets (high frequency) are translations with small steps to cover all time intervals, while wide (low frequency) wavelets are translational with wide steps. Because the width of $\psi\left(a_0^{-m}x\right)$ is proportional to the value $a=a_0^m$, discretization of the value of b can be chosen with $b=nb_0a_0^m$ and for some $b_0>0$ and integer n. So that a discrete wavelet transformation is given by

$$\psi_{m,n}(t)=a_o^{-m/2}\ \psi(a_o^{-m}\ [t-nb_0a_0^m]) \qquad (6)$$

where $a=a_0^m$, $a_0>1$ and $b=nb_0a_0^m$ and for some $b_0>0$ and integer n.

2.1.2 Haar, Daubechies, and Symlets wavelets

Wavelet Haar is the simplest type of wavelet that Alfred Haar proposed in 1909 as a special function. The Haar function is given by Equation (7).

$$\psi(x)=\begin{cases}1, & 0\leq x<1/2\\ -1, & \frac{1}{2}\leq x<1\\ 0, & \text{others}\end{cases}, \qquad (7)$$

with graphs as in Figure 2.

Daubechies wavelet is one type of orthogonal wavelet that is very popular for digital signal processing. This wavelet type was developed by Inggrid Daubechies in 1990. Unlike the Haar wavelet, Daubechies wavelets have several variations which are characterized as an order of Daubechies wavelets, known as Daubechies wavelets with an N-th order (DbN), for some natural number N. Especially for N = 1 or Db1, Daubhecies wavelet is Haar wavelet. Wavelet Daubechies order $N\geq 2$, has 2N vanishing moment and has compact support at interval [0, 2N-1]. The N-th order of Daubechies wavelets is related to the Daubechies polynomial order.

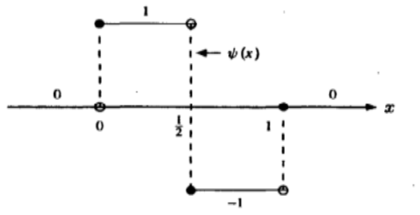

Figure 2: Graph of Haar wavelet

Daubechies polynomial order N-1 is defined as follows:

$$P_{N-1}(y)=\sum_{k=0}^{N-1}\binom{2N-1}{k}y^{k}(1-y)^{N-1-k} \qquad (8)$$

Graphically the Daubechies scaling and wavelet functions built by Daubechies polynomial N-th order, N = 2, 3,4, ..., 8 are given by Figure 3.

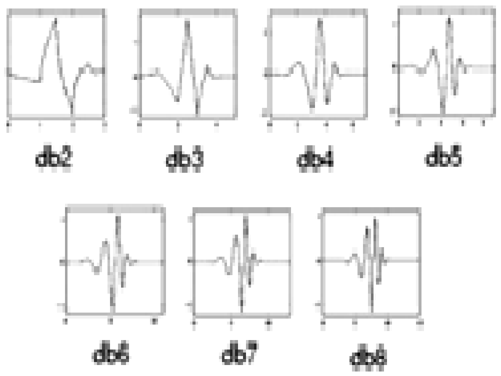

Figure 3: The graph variation of types of Daubechies wavelet i^{th} order, $i = 2,3,\cdots,8$.

Symlet Wavelet is a modification of Daubechies wavelet which has almost symmetrical characteristics, with the higher character the higher the symmetry quality. As a result of the modification of Daubechies wavelets, Symlet and Daubechies wavelets have similar characteristics.

The graphs from the Symlet wavelet for some *i-th* order are given in Figure 4. Like the Daubechies wavelet, the Symlet order 1 (Sym1) wavelet is also the Haar wavelet.

2.3 Wavelets and Approximation of Function

Suppose $f \in L^2[a,b]$ and $\int_a^b f^2(t)\,dt < \infty$. The wavelet representation of a function f is given by

$$f = c_0 + \sum_{j\geq 0}^{\infty}\sum_{k=1}^{2^j}\left\langle f,\psi_{j,k}\right\rangle \psi_{j,k}\,, \qquad (9)$$

where a c_0 constant,

$$c_0 = \left(\int_0^1 f(t)\,dt\right) \qquad (10)$$

and

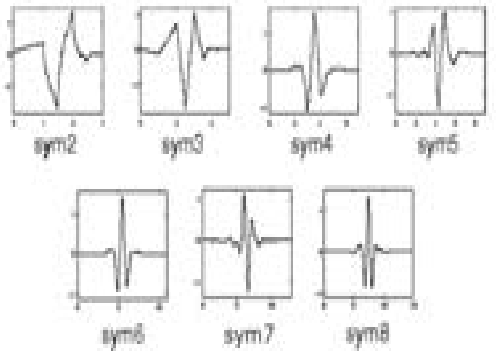

Figure 4: The graph variation of types of Symlet wavelet i^{th} order, $i = 2,3,\cdots,8$.

$$\psi_{j,k} = \left\langle f,\psi_{j,k}\right\rangle = \int_0^1 f(t)\psi_{j,k}(t)\,dt \qquad (11)$$

with orthonormal base functions $\psi_{j,k}$, associated with scale 2^{-j} and position $k2^{-j}$. Functions $\psi_{j,k}$ are called wavelets of scale 2^{-j} and position $k2^{-j}$.

3 RESULTS AND DISCUSSION

The use of wavelets to estimate nonlinear functions is a relatively new method. In this study, two types of wavelets Daubechies and Symlets are used to show the advantages of wavelets in approximating nonlinear functions applied to three sample data as nonlinear function representation such as dynamical system data, chaotic Mackey-Glass data, and hydrological data.

3.1. Dynamical System Data

In this example, the data used are dynamically generated system data based on the iteration equation (Banakar and Azeem, 2006):

$$x(n+1)=\frac{5x(n)}{1+x(n)^2}-0{,}5x(n)-0{,}5x(n-1)+0{,}5x(n-2), \qquad (12)$$

with the initial state $x(0)=0.2$, $x(1)=0.3$, and $x(2)=1$.

The approximation of nonlinear iteration function (12) uses wavelet approximation based on Daubechies and Symlets type wavelets in various orders (1-st, 2-nd, 3-rd, 4-th and 8-th order) on level 3-rd respectively given by Figure 5 and Figure 6.

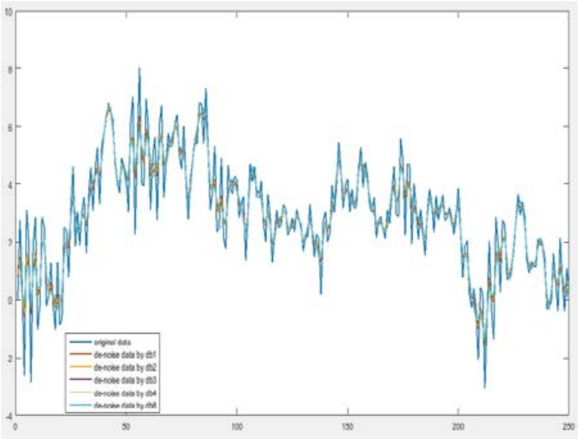

Figure 5: Original data (dynamical system example) versus approximated data using wavelet denoised with several orders of Daubechies wavelet.

Based on Figure 5 and 6, the approximation of the nonlinear function represented by Equation (12) using several orders of the Daubechies wavelet level 3-rd gives the result that the order 1-st of the Daubechies has the smallest MSE, that is 0.401 and the 8-th order of the Daubechies has the largest MSE that is 0.63856. For the same case, the the best approximation by Symlets wavelet type is given by the 1-st order with value of MSE is 0.401 and the worst approximation is given by the 8-th order with value of MSE is 0.62875.

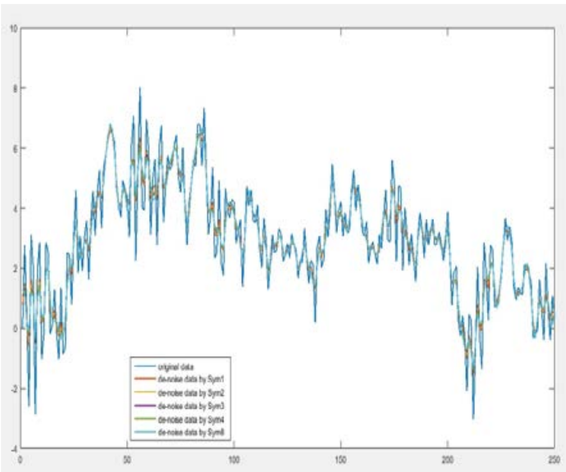

Figure 6: Original data (dynamical system example) versus approximated data using wavelet denoised with several orders of Symlets wavelet.

3.2 Chaotic Mackey-Glass Data

In this example, the data used is data generated based on the Mackey-Glass differential delay equation given by the Equation (13) (Banakar and Azeem, 2006).

$$\dot{x}(n+1) = \frac{0,2\,x(t-\tau)}{1+x^{10}(t-\tau)} - 0,1\,x(t) \qquad (13)$$

with the initial state $x(0) = 1.2$, $\tau = 17$, and $x(t) = 0$, for $t < 0$.

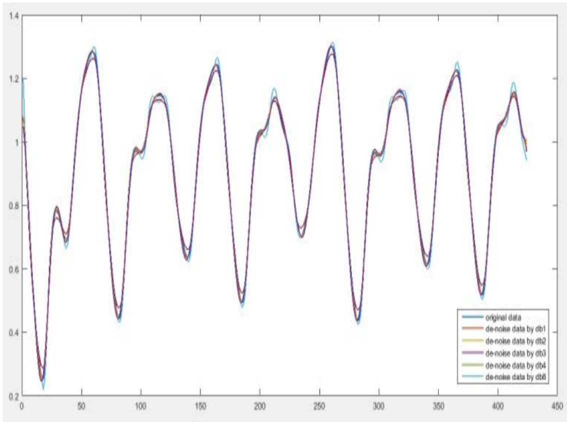

Figure 7: Original data (chaotic Mackey Glass) versus approximated data using denoised wavelet with several orders of Daubechies wavelet.

The approximation of nonlinear differential delay Equation (13) uses wavelet approximation based on Daubechies and Symlet type wavelets in various orders (1-st, 2-nd, 3-rd, 4-th and 8-th order) level 3-rd respectively given by Figure 7 and 8.

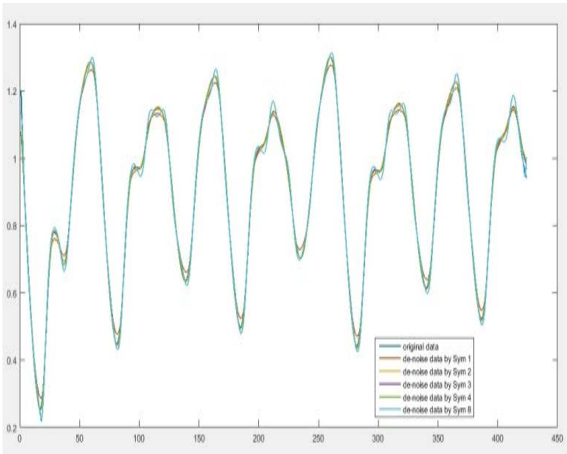

Figure 8: Original data (chaotic Mackey Glass) versus approximated data using denoised wavelet with several orders of Symlets wavelet.

Figure 7 and 8 establish for the Chaotic Mackey-Glass case that the approximation of nonlinear functions using Daubechies and Symles wavelets provides very good results based on the MSE indicator. The approximation using Daubechies and Symlets wavelets type with several orders $j = 1, 2, 3, 4,$ and 8 gives the result that the best approximation is obtained in the 2-nd order with the value of MSE is 2.8566×10^{-4}. For the same case, the value of MSE of the worst approximator using by Daubechies wavelet type is 4.2149×10^{-4} from 8-th order and 4.2058×10^{-4} using by Symlets wavelet type from 1-st order.

3.3 Hydrological Data

In this case, the data used is the data (daily) of water discharge of Sungai Ancar, Mataram Region, Lombok NTB period 2014-2016 (Source: Balai Nusa Tenggara I River Region). The approximation of the nonlinear function represented by debit of the Ancar River data using the wavelet approximation based on Daubechies and Symlet types in various orders (1-st, 2-nd, 3-rd, 4-th and 8-th order) level 3-rd respectively is given by Figure 9 and 10.

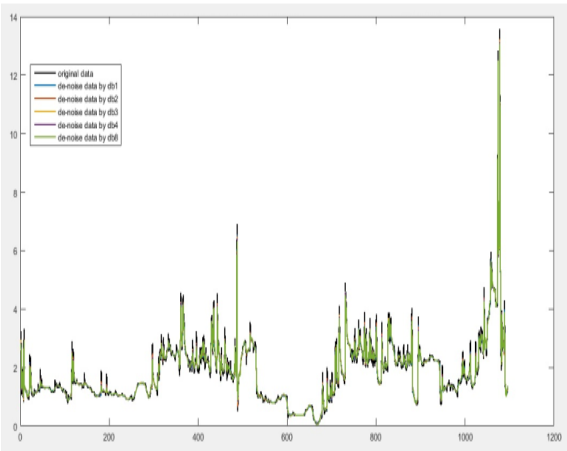

Figure 9: Original data (daily hidrology data) versus approximated data using wavelet denoised with several orders of Daubechies wavelet.

Based on three case examples, the performance of approximation functions based on Daubechies and Symlets type wavelet functions on actual nonlinear functions (explicit functions) or nonlinear function representations based on data on each given case sample based on variations in Daubechies and Symlet wavelet orders is given by Table 1.

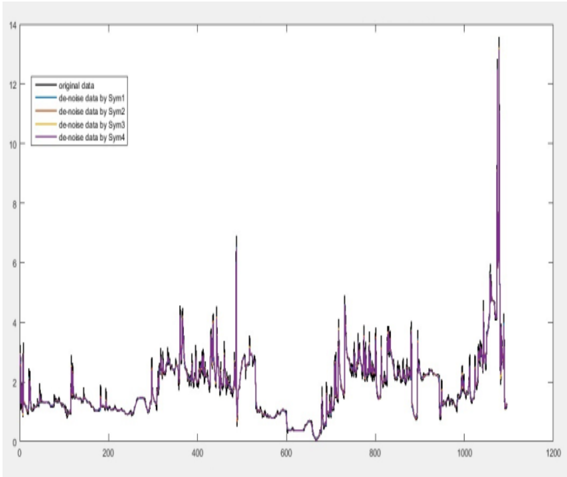

Figure 10: Original data (daily hidrologicals data) versus approximated data using wavelet denoised with several orders of Symlets wavelet.

For the daily hydrological data, Figure 9 and 10 show that the approximation of nonlinear functions using Daubechies and Symles wavelets type provides a good results based on MSE value indicators. The best approximation of the nonlinear function for this case provide the value of MSE is 1.4891×10^{-2} for Daubechies wavelet type from Db1 and 7.7689×10^{-3} for Symlets wavelet type from Sym8. But, for the worst approximators, Daubechies wavelets type given by Db8 with MSE values equal to 1.9411×10^{-2} and Symlets wavelet type given by Sym4 with value of MSE equal to 1.7508×10^{-2}.

Table 1: The value of mean square error (MSE) of data with various orders of Daubechies and Symlets wavelet.

Wavelets Type		Accuration of approximation with MSE indicator		
		Dynamical System Data	Chaotic Mickey-Glass Data	Daily Hydrologcal Data
Daubechies	Db1	0.40100	4.2058×10^{-4}	1.4891×10^{-2}
	Db2	0.46682	2.8566×10^{-4}	1.6440×10^{-2}
	Db3	0.51618	3.4148×10^{-4}	1.7243×10^{-2}
	Db4	0.55421	3.6877×10^{-4}	1.7739×10^{-2}
	Db8	0.63856	4.2149×10^{-4}	1.9411×10^{-2}
Symlets	Sym1	0.40100	4.2058×10^{-4}	1.4891×10^{-2}
	Sym2	0.46682	2.8566×10^{-4}	1.6440×10^{-2}
	Sym3	0.51618	3.4148×10^{-4}	1.7243×10^{-2}
	Sym4	0.54878	3.4460×10^{-4}	1.7508×10^{-2}
	Sym8	0.62875	3.4460×10^{-4}	7.7689×10^{-3}

Furthermore, based on Table 2 it can be seen that for the three case samples observed, the first two examples of dynamical system data and chaotic Mackey-Glass data obtained the same results. For dynamical system data, the best accuracy is given by the same order of the two types of wavelets, namely Db1 and Sym1 with the same level of accuracy, which is equal to MSE = 0.401. Similar to the chaotic Mackey-Glass data, the best accuracy is given by the same order, ie. Db2 and Sym2 with the same level of accuracy, which is equal to MSE = 2.8566×10^{-4}.

Table 2: Best accuration of the approximation with Daubechies and Symlets wavelets

Examples Case	Best Accuration with Wavelets Type			
	Daubechies Wavelet		Symlets Wavelet	
	Type	MSE	Type	MSE
Dynamical System Data	Db1	0.40100	Sym1	0.40100
Chaotic Mickey Glass Data	Db2	2.8566×10^{-4}	Sym2	2.8566×10^{-4}
Daily Hydrological Data	Db1	1.4891×10^{-2}	Sym8	7.7689×10^{-3}

For the daily hydrological data, the best accuracy for the Daubechies wavelet type is generated by the first order Daubechies wavelet (db1 or Haar wavelet) with accuracy (MSE) of 1.4891×10^{-2}. For Symlets wavelet type, the best accuracy is given by the Symlet 8-th order wavelet with an accuracy of 7.7689×10^{-3}.

4 CONCLUSIONS

The simulation results of the approximation of the nonlinear function using the Daubechies and Symlets wavelet type provide a fairly good accuracy based on the mean square error (MSE) indicator. The performance of two wavelets base for the first two cases, dynamical system and chaotic Mackey-Glass data, shows that the two wavelet bases with the same order provide the same level of accuracy. But, for the case of nonlinear function represented by real data, the debit of Ancar River data, the approximation function based on Daubechies wavelet is given by 1-st order (Db1), while for Symlets wavelet type is given by 8-th order (sym8).

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REFERENCES

Ahamada, I., and Jolivaldt, P., 2010, *Classical vs wavelet-based filters Comparative study and application to business cycle*, Document de Travail du Centre d'Economie de la Sorbonne - 2010.

Alexsandrov, T., Bianconcini, S., Dagum, E.B., Maass, P., and McElroy, T., 2008, *A Review of Some Modern Approaches to the Problem of Trend Extraction*, Research Report Series, Report Issued: March 28, 2008.

Alves, M.A., P. Cruz, A. Mendes, F.D. Magalhaes, F.T. Pinho, and P.J. Oliveira, 2002, Adaptive multiresolution approach for solution of hyperbolic PDEs, *Comput. Methods Appl. Mech. Engrg.*, 191, p. 3909 – 3928.

Bahri, S., Widodo, dan Subanar, 2016a, Applied Multiresolution B-Spline Wavelet to Neural Network Model and Its Application to Predict some Economis Data, *International Journal of Applied Mathematics and Statictics*, Vol. 54; Issue No. 1, p. 92-103.

Bahri, S., Widodo, dan Subanar, 2016b, Optimization of Wavelet Neural Networks Model by Setting the Weighted Value of Output Through Fuzzy Rules Takagi-Sugeno-Kang (TSK) TypeAs a Fixed Parameter, *Global Journal of Pure and Applied Mathematics*, Volume 12, Number 3, p. 2591–2603.

Bruzda, J., 2004, Wavelet vs. Spectral Analysis of an Economic Process, *Dynamic Econometric Models*. Vol. 6 – Nicolaus Copernicus University – Toruń – 2004.

Chen, P., 2002, *Understanding Economic Complexity and Coherence: Market Crash, Excess Capacity, and Technology Wavelets*, Paper which Presented at Shanghai International Symposium on Complexity Science.

Daubechies, Ingrid, 1992, *Ten Lectures on Wavelets. Philadelphia*, Society for Industrial and Applied Mathematics (SIAM).

Gallegati, M. and M. Gallegati, 2005. *Wavelet variance and correlation analyses of output in G7 countries.* JEL codes: C01, E31, E32. Online Paper. Diakses: 14 -2-2011.

Lineesh, M. C., Minu, K.K., and John, C.J., 2010, Analysis of Nonstationary Nonlinear Economic *Time series* of Gold Price A Comparative Study. *International Mathematical Forum*, 5, 2010, no. 34, p. 1673 – 1683.

Liu, Y. and Din, K., 2016, Research Article : A Numerical Method Based on Daubechies Wavelet Basis and B-Spline Patches for Elasticity Problems, Hindawi Publishing Corporation, *Mathematical Problems in Engineering*, Volume 2016, Article ID 2549213, http://dx.doi.org/10.1155/2016/2549213.

Matsumoto, Y. and J. Watada, 2007, Wavelet Approach to Chaotic Forecasting of Stock Movement. *Asia Pacific Journal of Finance and Banking Research* Vol. 1. No. 1.

Misiti, Michael, Yves Misiti, G. Oppenheim, and JM. Poggi, 2007, *Wavelets and their Applications*, USA: ISTE Ltd.

Yadav, T. and Mehra, R., 2016, Denoising ECG Signal Using Daubechies and Symlet Wavelet Transform Techniques, *International Journal of Advanced Research in Computer and Communication Engineering*, Vol. 5, Issue 9, p. 438-443.

Ysusi, C., 2009, Analysis of the Dynamics of Mexican Infation Using Wavelets, *Working Paper.*

Zainuddin, Z. and Pauline, O., 2011, Modified wavelet neural network in function approximation and its application inprediction of time-series pollution data, *Applied Soft Computing*, vol. 11, p. 4866–4874.