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The Bridge Graphs Partition Dimension

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The Bridge Graphs Partition Dimension

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Abstract. Finding the partition dimension of graph is still an open problem in the graph theory. Therefore, several researchers investigate the problem in several operations of the graph. For example, the partition dimension of corona product, cartesian product, subdivision operation has been published by several researchers. Let G_1 ,G_2 be two connected graphs. We present the partition dimension of a graph G which is obtained from two graphs G_1, G_2 with a linking a vertex in G_1 to one vertex in G_2 (a bridge in graph G). This paper is devoted to find the upper bound of partition dimension of the connected two graphs by a bridge.

Keywords: partition dimension, bridge graphs

1. Introduction

Several applications of dimensional concepts have been published by several studies such as navigation robots [7]. In addition, this concept can also be used in the optimization of the placement of threat detection sensors [6]. However, the problems with the concept of dimension graph have not yet been resolved such as the partition dimension of the bridge graph.

Suppose $G = (V, E)$ is a connected graph and x, y are two distinct vertices of G. The length of shortest path from a vertex x to vertex y is called the *distance* of x to y, denoted by $d(x, y)$. The distance from x to a subset $B = \{b_1, b_2, \cdots, b_k\}$ of $V(G)$ is min $\{d(x, b_i)|b_i \in A\}$. Suppose $B = \{L_1, L_2, L_3, \cdots, L_k\}$ is a partition of $V(G)$. The vector $\big(d(x, L_1), d(x, L_2), \cdots, d(x, L_k)\big)$ is called The *representation* of a vertex x with respect to the partition B, denoted by $r(x|B)$. For any distinct pair $x, y \in V(G)$, if $r(x|B) \neq r(y|B)$, then the partition B is called a *resolving partition* of G. The smallest cardinality of any resolving partition of G is called the $partition\ dimension$ of G, denoted by $pd(G)$. Two distinct x and y are called the distinguished by a class partition L_t for some $t \in [1, k]$, if the distance from u to L_t is different to distance from v to L_t. Suppose two distinct vertices u and v that are distinguished by L_t for some $t \in [1, k]$. So, If a vertex $x \in L_t$ is a vertex that has the nearest distance from u to L_t and a $y \in L_t$ is a vertex that has the nearest distance from y to L_t, then the vertices x and y in L_t are called the *distance defining vertices* of u and v in L_t.

Some researchers investigate the partition dimensions based on the certain clasess such as homogeneous firecrackers [4], series parallel graphs [10] and books graph [11[. The other researchers using the operations of graph such as corona [7, 13], subdivision [1,2,3,5], and cartesian operations [12]. In this case, we examine the operation of union of two graphs that linked by a bridge.

Suppose H_1, H_2 are two connected graphs, $u \in V(H_1)$ and $v \in V(H_2)$. A new graph G which is obtained from H_1 and H_2 with connecting a vertex u to v by a new edge uv is called the bridge graph, is denoted by $B(H_1, H_2, uv)$. If the vertex v is only adjacent to one vertex then the vertex v is called *a leaf.*

2. Basic Concepts

In this section, we give some useful concepts to determine our results. The following Lemmas 2.1 shows the properties of two vertices that have the same distance to any other vertices of the graph.

Lemma 2.1 [9]
Suppose B is a resolving partition of a connected graph G. If the distance from u to w is the same to distance from v to w for all vertices w in G except u and v, then the vertex u is in a different partition class of B than the vertex v
By this lemma, we know that the graph has a vertex with k leaves, it has partition dimension at least k. So, the Lemma 2.1 gives minimum of partition dimension of the graph with a vertex which is adjacent to k leaf.
Corollary 2.1 [9]
Suppose H is a connected graph and k is an integer, the lower bound of partition dimension of H is at least k, if the graph H has a vertex v that is adjacent to k leaves.
Lemma 2.2 [8]
The graph H is a path P_n where $n \geq 2$ if only if the partition dimension of H is two.
The Lemma 2.2 show that the partition dimension of any connected graph and non-trivial is at least three except for a path. Some other certain graphs have obtained the partition dimension such as complete graph K_n, star graph $K_{1,n}$ and cycle graph C_n where $pd(K_n) = n, pd(K_{1,n}) = n$, and $pd(C_n) = 3$.

3. The Main Results

This section, we give our main results. On the Theorem 3.1 until 3.5, we give the partition dimension of $B(G_1, G_2, e)$ for $G_i, i \in 1,2$ is a special graph. The Theorem 3.6 give the tight upper bound of partition dimension of $B(G_1, G_2, e)$ for any connected graph G_i , and $i \in 1,2$.
First, in Theorem 3.1 we give the partition dimension $B(G_1, G_2, e)$ f where $G_i, i \in 1,2$ is a path.
Theorem 3.1
If P_m, P_n are two paths for $m, n \geq 2$, and $u \in V(P_m), v \in V(P_n)$, then

$$pd\big(B(P_m, P_n, uv)\big) = \begin{cases} 2 & u, v \textit{ are the leaves} \\ 3 & \textit{othewise} \end{cases}$$

Proof
Consider u, v in $B(P_m, P_n, uv)$. If the vertices u, v are the leaves, then $B(P_m, P_n, uv)$ is a path. By Theorem 2.3, $pd\big(B(P_m, P_n, uv)\big) = 2$.
If vertices u, v is the other, then we have that $B(P_m, P_n, uv)$is not a path. So, we have $pd\big(B(P_m, P_n, uv)\big) \geq 3$.
Now, we will show $pd\big(B(P_m, P_n, uv)\big) \leq 3$.
Let $V(P_m) = \{u_1, u_2, \cdots, u_m\}$ and $V(P_n) = \{v_1, v_2, \cdots, v_n\}$.
Let $uv = u_p v_q$ be a bridge of $B\left(P_m, P_n, u_{pv_q}\right)$ for some $p \in [1, m]$ and $q \in [1, n]$. Let $v_i \in L_2$ and $v_j \in M_2$. Define a new partition $L = L_1, L_2, L_3$ of $V\big(B(P_1, P_2)\big)$ where $L_1 = u_1, u_2, \cdots, u_p$, $L_2 = \{u_{\{p+1\}}, u_{\{p+2\}}, \cdots, u_{\{m\}}\} \cup \{v_1, v_2, \cdots, v_{q-1}\}$ and $L_3 = \{v_q, v_{q+1}, \cdots, v_n\}$, see at the Figure 1.
If $x, y \in L_1$ or $x, y \in L_3$, then they are distinguished by L_2. If $x, y \in L_2$, then they are distinguished by M_2 or M_3. This show that the partition Π is a resolving partition of $B(P_m, P_n, uv)$. So, we have $pd\big(B(P_m, P_n, uv)\big) = 3$.
■

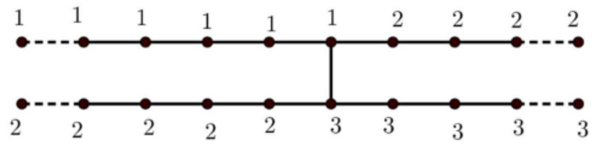

Figure 1. $pd\big(B(P_m, P_n)\big) = 3$.

Theorem 3.2
If C_m, C_n are two cycles, then $pd\big(B(C_m, C_n, uv)\big) = 3$ for $u \in V(C_m), v \in V(C_n)$.
Proof
Let $V(C_m) = \{u_1, u_2, \cdots, u_m\}$ and $V(C_n) = \{v_1, v_2, \cdots, v_n\}$. Let $u_1 v_1$ be a bridge of $B(C_m, C_n, u_1 v_1)$. Define $L = \{L_1, L_2, L_3\}$ is a partition of $V\big(B(P_1, P_2, u_1 v_1)\big)$where $L_1 = \{u_1, u_2, \cdots, u_{\{m-1\}}\} \cup \{v_n\}$, $L_2 = \{v_{n-1}\}$ and $L_3 = \{v_1, v_2, \cdots, v_{n-2}\} \cup \{u_m\}$.
For $z, w \in L_1$, if $d(z, L_3) = d(w, L_3)$ then the vertices z, w are distinguished by v_{n-1}. If the distance from z to L_3 is different to distance from w to L_3 then they are distinguished by L_3. For $u, w \in L_3$, if $d(z, L_1) = d(w, L_1)$ then z, w are distinguished by u_{n-1}. The otherwise, they are distinguished by L_1. We know that the partition class L_2 contains only one vertex. So, these show that partition L is a resolving partition of $B(C_m, C_n, uv)$. Thus, we obtain $pd\big(B(C_m, C_n, uv)\big) = 3$.
Theorem 3.3
Suppose G is a connected graph and K_n is a complete graph where $n \geq 4$. If $u \in V(G)$ and $v \in V(K_n)$, then $pd\big(B(G, K_n, uv)\big) \geq n$.
Proof
Let $V(K_m) = \{v_1, v_2 \cdots, v_n\}$. Let $v = v_1$ and T be a resolving partition of $B(G, K_n, uv_1)$. Since $v_2, v_3 \cdots, v_n$ have the same distance to others vertices of $B(G, K_n, uv_1)$, then they are in the distinct partition classes of T. Base on the Lemma 2.1, we have $pd\big(B(K_m, K_n, uv_1)\big) \geq n - 1$.
Suppose the partition $T = \{L_1, L_2, \cdots, L_{n-1}\}$ is a resolving partition of $B(G, K_n, uv_1)$. Now consider v_1 in partition class of T. Since T is a resolving partition of $B(G, K_n, uv_1)$, the vertex v_1 must be in L_i for some $i \in [1, n-1]$. Without loss of generality (w.l.o.g), let $v_1 \in L_1$. This implies $v_1, v_j \in L_1$ for some $j \in [2, n]$. Thus, we obtain $d(v_1, L_t) = d(v_j, L_t)$ for $t \in [1, n-1]$. So, we have $r(v_1|L) = r(v_j|L)$, contradiction. This implies $pd\big(B(K_m, K_n, uv_1)\big) \geq n$.
Theorem 3.4
If K_m, K_n be two complete graph with $3 \leq m \leq n$ and $v \in V(K_m)$, $w \in V(K_m)$, then $pd(B(K_m, K_n, vw) = n$
Proof
Let $V(K_m) = \{v_1, v_2, \cdots, v_m\}$, $V(K_n) = \{w_1, w_2, \cdots, w_n\}$ and $vw = v_1 w_1$ be a bridge of $B(K_m, K_n, v_1 w_1)$.
By Lemma 3.3, we have $pd\big(B(K_m, K_n, v_1 w_1)\big) \geq n$.
Now, let $n \geq m$. Let the partition $T = \{M_1, M_2, \ldots, M_n\}$ be a partition of $V\big(B(K_m, K_n, v_1 w_1)\big)$ where $M_1 = \{v_1, v_m, w_1\}$, $M_i = \{v_i, w_i\}$ for $2 \leq i \leq m-1$, $M_i = \{w_i\}$ for $m \leq i \leq n$.
Let x, y be two different vertices which are in the same partition class of T.
For $x, y \in M_i$ with $1 \leq i \leq m-1$, the vertices x, y are distinguished by M_n. So, this show that $r(x|\Pi) \neq r(x|\Pi)$. For $x, y \in M_i$ with $m \leq i \leq n$, Clearly M_i is a singleton. Thus, T is a resolving partition of $B(K_m, K_n, v_1 w_1)$. This implies $pd\big(B(K_m, K_n, v_i w_1)\big) = n$.
The next theorem, we give the partition dimension of star graph $K_{1,n}$. The star graph $K_{1,n}$is a complete bipartite graph where $V\big(K_{1,n}\big) = \{v, v_1, v_2, \cdots, v_n\}$and $E\big(K_{1,n}\big) = \{vv_i | 1 \leq i \leq n\}$. The vertex v which is adjacent to n other vertices is called a *center vertex* of $K_{1,n}$.
Theorem 3.5
Let $K_{1,m}, K_{1,n}$ be two star graphs with $2 \leq m \leq n$.
If $z \in V\big(K_{1,m}\big)$and $w \in V\big(K_{1,n}\big)$ then

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$$pd\left(B(K_{1,m},K_{1,n},zw)\right)=\begin{cases} n-1 & if\ z,w\ are\ two\ leaves\ or \\ & z\ is\ a\ leaf\ and\ w\ is\ a\ center\ vertex\ and\ n>m \\ n & otherwise\end{cases}$$

Proof

Let $V(K_{1,n})=\{v,v_1,v_2,\cdots,v_n\}$, where v is a center vertex of $K_{1,n}$. Let $V(K_{1,m})=\{u,u_1,u_2,\cdots,u_m\}$, where u is a center vertex of $K_{1,m}$.

Consider z,w in two cases.

Case 1), for z,w are two leaves, and let $z=v_1$ and $w=u_1$.

Let $L=\{L_1,L_2,\cdots,L_{n-1}\}$ be a resolving partition of $B(K_{1,m},K_{1,n},u_1v_1)$where $L_1=\{v_1,v_n,u_2\}$, $L_2=\{u_1,u,v_2\}$, $L_i=\{v_i,u_{i+1}\}$ for $3\le i\le m-1$, $L_i=\{v_i\}$ for $m\le i\le n-2$, and $L_{n-1}=\{v,v_{n-1}\}$. Easily to show that L is a resolving partition of $B(K_{1,m},K_{1,n},u_1v_1)$. This implies that $pd\left(B(K_{1.m},K_{1,n},zw)\right)\le n-1$.

Now, consider graph $B(K_{1,m},K_{1,n},u_1v_1)$. We know that $B(K_{1,m},K_{1,n},u_1v_1)$ has a vertex v with $n-1$ leaves. So, by Corollary 2.1, we have $pd\left(B(K_{1,m},K_{1,n},zw)\right)\ge n-1$. These implies that $pd\left(B(K_{1,m},K_{1,n},zw)\right)=n-1$.

Next, if $w=v_1$ is a leaf and $z=u$ is a center vertex of $K_{1,m}$ and $n>m$ then the graph $B(K_{1,m},K_{1,n},zw)$ has a vertex v which is adjacent to $n-1$ leaves. So, by Corollary 2.1, we have $pd\left(B(K_{1,m},K_{1,n},zw)\right)\ge n-1$. Let $L=\{L_1,L_2,\cdots,L_{n-1}\}$ be a resolving partition of $B(K_{1,m},K_{1,n},u_1v_1)$ where $L_1=\{v_1,v_2,u_1\}, L_2=\{u,u_2,v_3\}, L_i=\{v_{i+1},u_i\}$ for $3\le i\le m$, $L_i=\{v_i\}$ for $m\le i\le n-2$, and $L_{n-1}=\{v,v_{n-1}\}$. Easily to show that L is a resolving partition of $B(K_{1,m},K_{1,n},zw)$. This implies that $pd\left(B(K_{1,m},K_{1,n},zw)\right)=n-1$.

Case 2), if $w=v_1$ is a leaf and $z=u$ is a center vertex of $K_{1,m}$ and $n=m$ then the graph $B(K_{1,m},K_{1,n},zw)$ has a vertex v which is adjacent to n leaves. So, by Corollary \ref{coro1}, we have $pd\left(B(K_{1,m},K_{1,n},zw)\right)\ge n$.

Let $L=L_1,L_2,\cdots,L_n$ be a resolving partition of $B(K_{1,m},K_{1,n},u_1v_1)$,) where $L_1=\{u_1,v_1,v_2\}$, $L_2=\{u,u_2,v_3\}$, $L_i=\{v_{i+1},u_i\}$ for $3\le i\le n-2$, $L_{n-1}=\{v,v_n,u_{n-1}\}$, and $L_n=\{u_n\}$. Easily to show that L is a resolving partition of $B(K_{1,m},K_{1,n},zw)$. This implies that $pd\left(B(K_{1,m},K_{1,n},zw)\right)=n$.

Next, if $w=v$ and $z=u$ is a center vertex of $K_{1,n}$ and $K_{1,m}$, then the graph $B(K_{1,m},K_{1,n},zw)$ has a vertex v which is adjacent to n leaves. So, by Corollary 2.1, we have $pd(B(K_{1,m},K_{1,n},zw)\ge n$.

Let $L=\{L_1,L_2,\cdots,L_n\}$ be a resolving partition of $B(K_{1,m},K_{1,n},u_1v_1)$ where $L_1=\{u,u_1,v_1\}$, $L_2=\{v,v_2\}$,$L_i=\{v_i,u_i\}$ for $3\le i\le m$, $L_i=\{v_i\}$ for $m+1\le i\le n$. Easily to show that L is a resolving partition of $B(K_{1.m},K_{1,n},zw)$. This implies that $pd\left(B(K_{1.m},K_{1,n},zw)\right)=n$.

On the Lemma 3.1, we give a distance between a vertex u and a distance defining vertex u under a partition class of Π . In this case, we consider two distinct vertices u,v which have the same distance to a dominant vertex of G.

Lemma 3.1

Suppose G is a connected graph, $u,v,z\in V(G)$ and Π be a resolving partition of G. Let z be a dominant vertex under Π , and the vertices u and v have the same distance to z. Let x and y be two distance defining vertices of u and v.

If $d(u,x)<d(v,y)$, then $d(u,x)<d(u,z)$.

Proof

Clearly $x\ne z$ because if $x=z$, then $d(u,x)=d(v,x)$, a contradiction with x is a distance defining vertex of u. Let $\Pi=\{L_1,L_2,\cdots,L_k\}$ and $x,y\in L_t$ for some $t\in[1,k]$. Since z is a dominant vertex

under Π, there is a vertex w such that$wz \in E(G)$ and $w \in L_t$. If w is a internal vertex of a $u - z\, path$, i.e., $u \cdots wz$, then $d(u,w) = d(u,z) - 1$. Otherwise, we have $d(u,w) = d(u,z) + 1$.
If w is a internal vertex of a $v - z\, path$, then $d(v,w) = d(v,z) - 1$. Otherwise, we have $d(v,w) = d(v,z) + 1$. If w is a internal vertex of $u - z$ and $v - z\, paths$ then we have $d(u,x) < d(v,y) \leq d(v,w) < d(u,z)$. If w is a internal vertex of $u - z\, path$ and w is not a internal vertex of $v - z\, path$ then we have $d(u,x) \leq d(u,w) = d(u,z) - 1 < d(u,z)$ and $d(v,y) \leq d(v,w) = d(v,z) + 1$.
If w is not a internal vertex of $u - z$ path and w is a internal vertex of $v - z\, path$ then we have $d(v,y) \leq d(y,w) = d(u,z) - 1 < d(u,z)$. Since $d(u,x) < d(v,y)$, we have $d(u,x) < d(u,z)$. If w is not a internal vertex of $u - z\, path$ and $v - z\, path$ then we have $d(v,y) \leq d(v,w) = d(v,z) + 1$. Since $d(u,x) < d(v,y)$, we have $d(u,x) < d(v,y) \leq d(v,w) = d(v,z) + 1 = d(u,z) + 1$. So, $d(u,x) < d(u,z) + 1$. Since $x \neq z$, we obtain $d(u,x) < d(u,z)$.

Theorem 3.6

Let G_1, G_2 be a connected graphs, $L = L_1, L_2, \cdots, L_s$ and $M = \{M_1, M_2, \cdots, M_k\}$ be two resolving partition of G_1 and G_2. Let $a \in V(G_1)$ and $b \in V(G_2)$. If a$\in V(G_1)$, $b \in V(G_2)$ be two dominant of G_1 and G_2 under L and M then $pd\big(B(G_1, G_2, ab)\big) \leq max\{s,k\} + 1$.

Proof

Let $a \in L_1$ and $b \in M_1$. Let $k = max\{s,k\}$. Define a new partition $\Pi = \{P_1, P_2, \cdots, P_{k+1}\}$ of $B(G_1, G_2, ab)$ where $P_i = L_i \cup M_i$ for $i \in [2,s]$, $P_1 = L_1, P_{k+1} = M_1$, and $P_i = M_i\, for\, s + 1 \leq i \leq k$. We will show that Π is a resolving partition of $B(G_1, G_2, ab)$.
Let u, v be two distinct vertices of $B(G_1, G_2, ab)$. If $u \in V(G_1) and v \in V(G_2)$, then consider $d(u,a)$ and $d(v,a)$. If $d(u,a) \neq d(v,a)$, then u, v are distinguishing by $P_1\, or\, P_{k+1}$.
If $d(u,a) = d(v,a)$,then we have $d(u,P_1) \leq d(u,a)$, because $a \in P_1$. Consider two conditions $d(u,P_1) < d(u,a)$ or $d(u,P_1) = d(u,a)$. If $d(u,P_1) < d(u,a)$, then we obtain $d(u,P_1) < d(u,a) = d(v,a) = d(v,P_1)$ because $P_1 \subseteq V(G_1)$. So, the vertices u,v are distinguished by P_1. if $d(u,P_1) = d(u,a)$, then we obtain $d\big(u, P_{\{k+1\}}\big) = d(u,b) = d(u,a) + 1$ and $d\big(v, P_{\{k+1\}}\big) \leq d(v,b) = d(v,a) - 1$. This implies that u, v are distinguishing by P_{k+1}.
Next, we consider $u, v \in V(G_1)$ or $u, v \in V(G_2)$. Without loss of generality (w.l.o.g), let $u, v \in V(G_1)$. Since L is a resolving partition of G_1 , there is a partition class $L_q, 1 \leq q \leq s$ which is distinguishing the vertices u, v. Let x and y in L_q are a distance defining vertex of u and v. Let $d(u,x) < d(v,y)$. By Lemma 3.1, we have $d(u,x) < d(u,a)$. So, we obtain $d(u,P_t) = d(u,L_t)$.
Since $v \in V(G_1)$ and vertex a is a dominant vertex, there is at least one vertex $w \in L_q$ which is adjacent to a. So we have $d(u,x) < d(v,y) \leq d(v,w)$. This implies that x and y is too the distance defining vertices of u and v in $B(G_1, G_2, ab)$. Thus, the vertices u, v are distinguished by P_q. Therefore, we obtain that Π is a resolving partition of $B(G_1, G_2, ab)$. This implies $pd\big(B(G_1, G_2, ab)\big) \leq k + 1$.
Theorem 3.3 shows value of lower bound of the partition dimension of the graph. Whereas Theorem 3.6 give to upper bound of the partition dimension. So, easily to get the next Corollary 3.1.

Corollary 3.1

If $pd(G)$ is less than order of complete graph K_n with $n \geq 3$ and a vertex x is a dominant vertex of G, then $n \leq pd(B(G, K_n, xy) = n + 1$ for any connected graph G and $y \in V(K_n)$.

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