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The partition dimension of subdivision graph on the star

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Abstract. The partition dimension of the graphs is one of the open problems in graph theory. One of the methods which are used researcher is a graph operation, for example, subdivision operations. Let G be a connected graph of order n . The subdivision operation of G is an operation in G that replaces any edge by a path P_{k_i+2} for $k_i \geq 1$ and $i \in [1, n]$. The result graph of this operation is called the subdivision graph, is denoted by $S(G; k_1, k_2, \dots, k_n)$. Furthermore, if each $k_i = 1$, then the subdivision graph is denoted by $S(G)$. One of the most recent results on the partition dimension of subdivision operation in the general graphs is published by Amrullah, et al 2016. However, the results show only an upper and lower bounds of the subdivision graph partition dimension. Therefore, this paper is devoted to finding the partition dimension of subdivision graph $S(G)$ and $S(G; k_1, k_2, \dots, k_n)$ on special star graphs $G = K_{1,n}$.

1. Introduction

Application of partition dimension can be found in various fields such as network discovery and verification [1], robot navigation [2], the Djokovic-Winkler relation [3] and strategies for the Mastermind game [4]. However, the applications still are limited to use because there are many the graph that is unknown of its partition dimension. Therefore, the problem of partition dimensions is one of the interesting open problems in graph theory to investigate. The various researches have been investigated in diverse ways such as those based on certain graph classes, as well as the using graph operations. The using graph operations among others are corona operation [5], cartesian operation [6], comb product [7] and subdivision operation [8,9]. The subdivision operation on the graph, G , is an operation that obtaining a new graph with larger order than the order of G . In other words, if this subdivision operation is applied to determine the partition dimensions of the graph, then the partition dimension in the graph of the operation results can be obtained the base on the graph G .

Let G be a connected graph $A = \{e_1, e_2, \dots, e_t\}$. The *subdivision* of a graph G on the set A is denoted by $S(G; n_1, n_2, \dots, n_t)$ is a graph obtained from the graph G by replacing each edge e_i with a path P_{n_i+2} for $i \in [1, t]$. In particular, if $A = E(G)$ and $n_i = 1$ then the subdivision of graph G on A is simply denoted by $S(G)$. If $A = \{e\}$ and $n_1 = k$, then subdivision graph is denoted by $S(G(e, k))$. The internal vertices of the path replacing edge e are called *subdivision vertices*. For connected graf G with order n , the subdivision vertices of $S(G)$ are labeled by x_1, x_2, \dots, x_n . The research of partition dimension by subdivision operation The research on partition dimensions by subdivision operation is only several that was published such as complete graphs [9] and caterpillar graphs [8]. However, that researches have given only the partition dimension by subdivision operation on one edge $S(G(e, k))$ and all edges in one-time subdivision $S(G)$. Although the research of partition dimension of star graphs was



published in 2000 by [10]. However, no researches on the partition dimension of subdivision graph of star $K_{1,n}$ have been published until now. So, in this paper, we determine the partition dimension of $S(G)$ and $S(G(A, n_1, n_2, \dots, n_t))$ for a special graph G is star $K_{1,n}$.

2. Methods

On this section, we give several notations and basic concepts which will be used to find the partition dimension.

2.1. Basic Concepts

Let $G = (V, E)$ be a connected graph, and $u, v \in V(G)$. The definition of distance $d(u, v)$ from vertex u to vertex v is the length of a shortest path between u and v . Furthermore, we give a definition of distance from a vertex to subset of $L \subseteq V(G)$. Let $L = \{v_1, v_2, \dots, v_k\}$ be a subset of $V(G)$. The distance from a vertex v to L , $d(v, L)$, is $\min\{d(v, v_i) | v_i \in L\}$.

Next, we explain the concept of partition dimension of a connected graph G . Let $\Pi = \{L_1, L_2, L_3, \dots, L_k\}$ be a k -partition of $V(G)$. The representation $r(v|\Pi)$ of vertex v with respect to Π is the vector $(d(v, L_1), d(v, L_2), \dots, d(v, L_k))$. The partition Π is called a resolving partition of G if $r(w|\Pi) \neq r(v|\Pi)$ for all distinct $w, v \in V(G)$. The partition dimension of G , denoted by $pd(G)$, is a minimum cardinality of the resolving partitions of G . If two distinct vertices u, v have the different distance to some partition class L_j , $d(u, L_j) \neq d(v, L_j)$ for $j \in [1, k]$ then we say that u and v are distinguished by L_j or u and v are distinguishable. Let $v \in L_i$, if $d(v, L_j) = 1$ for any $L_j \neq L_i$ then v is called a dominant vertex under Π . Let L_t be a partition class distinguishing two vertices u, v where $t \in [1, p]$. Vertices x and y in L_t are called the distance defining vertices of u and v in L_t if $d(u, L_t) = d(u, x)$, $d(v, L_t) = d(v, y)$ and $d(u, x) \neq d(v, y)$.

2.2. The partition dimension properties of graphs

This following lemmas are very useful to finding the partition dimension of the graphs.

Lemma 2.1. [11] Let G be a connected with a resolving partition Π . If $d(u, w) = d(v, w)$ for all $w \in V(G) - \{u, v\}$, then vertices u, v must be in distinct partition classes of Π .

As direct consequence of Lemma 2.1, in the following corollary give a lower bound of the partition dimension of graphs.

Corollary 2.2. [11] Let G be a connected graph, if G has a vertex having k leaves then $pd(G) \geq k$.

The next Lemma 2.3 give the partition dimension of path P_n .

Lemma 2.3. [10] Let G be a connected graph of order $n \geq 2$. Then $pd(G) = 2$ if and only if $G = P_n$.

The Lemma 2.3 give the lower bound of partition dimension of others graph than a path.

Theorem 2.4. [10] Let G be a connected graph of order $n \geq 3$. Then $pd(G) = n - 1$ if and only if G is one of the graphs $K_{1,n-1}$, $K_n - e$, $K_1 + (K_1 \cup K_{n-1})$.

On the Theorem 2.4 give partition dimension of star graph $K_{1,n}$.

3. Results and Discussions

This section describes the partition dimension of the subdivision graph on any edge of the star graph $K_{1,n}$. In general, suppose G is a connected graph, $v \in V(G)$. We suppose that v is adjacent to u_1, u_2 , and let $e_1 = vu_1$, $e_2 = vu_2$. The subdivision graph on the both edges e_1 and e_2 is denoted by $S(G(e_1, e_2, 1, 1))$. Let x_1 and x_2 be the subdivision vertex on the edge e_1 and e_2 .

The following lemma explains that the subdivision vertices of x_1, x_2 must be in the distinct partition class of $S(G(e_1, e_2, 1, 1))$. This lemma is required to find partition dimension for all edge of star graph $S(K_{1,n})$.

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Lemma 3.1. Let G be a connected graph, $e_1 = vu_1, e_2 = vu_2 \in E(G)$ where u_1, u_2 are two leaves of G . Let $F = S(G(e_1, e_2, 1, 1))$ and Π be a resolving partition of F . If u_1, u_2 are in the one partition class of Π , then the subdivision vertices x_1, x_2 must be in distinct partition classes of Π .

Proof. Let $\Pi = \{L_1, L_2, \dots, L_p\}$ be a resolving partition of F and $v \in L_1$. Let $u_1, u_2 \in L_k$ for some $k \in [1, p]$. For a contradiction, suppose subdivision vertices $x_1, x_2 \in L_t$ for some $t \in [1, p]$. Since u_1, u_2 are in the same partition class, then we obtain that the distance from x_1 to L_i is equal to distance from x_2 to L_i for each $i \in [1, p]$. So, consequence, we have $r(x_1|\Pi) = r(x_2|\Pi)$, a contradiction.

In addition to the above lemma, the following lemma give the properties of leaves associated with the subdivision edges e_1, e_2 of the graph $F = S(G(e_1, e_2, 1, 1))$. In the following Lemma 3.2, we show that the vertex v and one of vertices u_1, u_2 can not be in the same partition class.

Lemma 3.2. Let G be a connected graph, $e_1 = vu_1, e_2 = vu_2 \in E(G)$ with u_1, u_2 be two leaves of G . Let $F = S(G(e_1, e_2, 1, 1))$, $\Pi = \{L_1, L_2, \dots, L_p\}$ be a resolving partition of F and $x_1, x_2 \in L_t$ for some $t \in [1, p]$. If $u_1 \in L_t$ and $v \in L_1$, then $u_2 \notin L_1$.

Proof. For a contradiction, suppose $u_2 \in L_1$. Since x_1, x_2 in the same partition class L_t , So we obtain that the distances of x_1, x_2 to each partition class L_i with $i \in [1, p]$ are the same. This implies $r(x_1|\Pi) = r(x_2|\Pi)$, a contradiction.

Let $K_{1,n}$ is a star graph, $V(K_{1,n}) = \{v\} \cup \{u_1, u_2, \dots, u_n\}$ and $E(K_{1,n}) = \{vu_i | 1 \leq i \leq n\}$. The vertex v is called a *center vertex* of $K_{1,n}$, and the others are called the *leaves*. On the Lemma 3.3 give condition of the partition dimension of graph subdivision in all edges of $S(K_{1,n})$.

Lemma 3.3. For $n \geq 5$, $pd(S(K_{1,n})) = p$ where p is smallest of positive integers satisfying $p(p-1) + 1 \geq n$.

Proof. The number p is a smallest of positive integer satisfying $p(p-1) + 1 \geq n$. So, we have $(p-1)((p-1)-1) + 1 = p^2 - 3p + 3 < n$. We will show that $pd(S(K_{1,n})) \geq p$ with $n \geq 5, p \geq 3$.

For a contradiction, suppose $\Pi = \{L_1, L_2, \dots, L_{p-1}\}$ is a resolving partition of $S(K_{1,n})$. Since there are $p-1$ partition classes of Π , and by Lemma 3.1, then there are $(p-1)(p-1)$ distinct pairs of the partition classes that are possible to n pairs x_i, u_i with $i \in [1, n]$. Without loss of generality, let $v \in L_1$. By Lemma 3.2 and $v \in L_1$, if $x_i, x_j, u_j \in L_k$ with $k \in [2, p]$ and $i, j \in [1, n]$, then $u_i \notin L_1$. This implies that there are $p-2$ possible pair of partition classes that can not be used for the verices x_i, u_i namely (x_i, u_i) are not in $(L_1, L_2), (L_1, L_3), \dots, (L_1, L_{p-1})$. Since there are $(p-1)(p-1)$ distinct pairs of partition classes that are possible for n pairs x_i, u_i and there are $p-2$ pair of partition classes that can not be used, then we have at most $(p-1)(p-1) - (p-2) = (p^2 - 2p + 1) - p + 2 = p^2 - 3p + 3$ partition classes which are used for n pair x_i, u_i . However we know $p^2 - 3p + 3 < n$. This show that there are s number of positive integer such that $p^2 - 3p + 3 + s = n$. This means that there are s vertices x_i, u_i can not belong in partition classes of Π , a contradiction to Π as a resolving partition of $S(G)$. So, we obtain $pd(S(G)) \geq p$.

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Let N be a set of non negative integer, $\Pi = \{L_1, L_2, \dots, L_p\}$ be a partition of $V(S(G))$ where $L_1 = \{v, x_1, x_2, \dots, x_p, u_1\}$, $L_i = \{u_t | t = 2 + j(p-1), j \in N\} \cup \{x_t | (i-1)p - (i-3) \leq t \leq i.p - (i-p)\} \cup \{u_t | t = j.p + (i-j), j \in N\}$, for $2 \leq i \leq p$, look at Figure 1. We will show that Π is a resolving partition of G . It shows that $pd(S(G)) \leq p$.

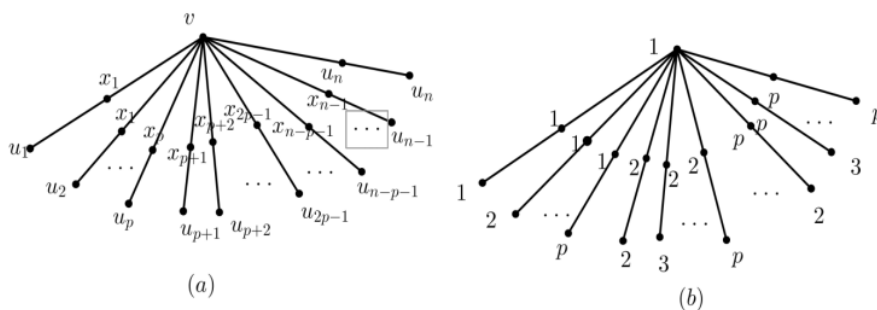


Figure 1. a). Subdivision graph $S(G)$ with $G = K_{1,n}$, b). The resolving partition of $S(K_{1,n})$.

Let x, y be two distinct vertices in the same partition class of Π . If $x = x_i$ and $y = x_j$ with $i \neq j$, then by definition of Π , we obtain that u_i and u_j must be in two distinct partition classes. This implies that x, y are distinguished by u_i or u_j . If $x = u_i$ and $y = u_j$, then x_i and x_j are in two distinct partition classes. So, we obtain that x, y are distinguished by x_i or x_j . If $x = x_i$ and $y = u_j$, then there is L_s for some $s \in [1, p]$, so we have $d(x, L_s) = 2$ and $d(y, L_s) = 3$. This implies that x, y are distinguished by L_s . If $(x = v)$ and (any vertex y is in L_1), then it is clearly that x is a dominant vertex and y is not a dominant vertex. Therefore, this shows that Π is a resolving partition of $S(G)$. So, we obtain $pd(S(G)) = p$.

Lemma 3.4. For $n = 3, 4$, $pd(S(K_{1,n})) = 3$.

Proof. Since $S(K_{1,3})$ is not a path, $pd(S(K_{1,n})) \geq 3$. The Figure 2 give the resolving partition of $S(K_{1,3})$ and $S(K_{1,4})$. So, we obtain Table 1 showing the representation of all vertices in $S(K_{1,4})$ and $S(K_{1,3})$. Therefore, we have $pd(S(K_{1,n})) = 3$ for $m = 3, 4$.

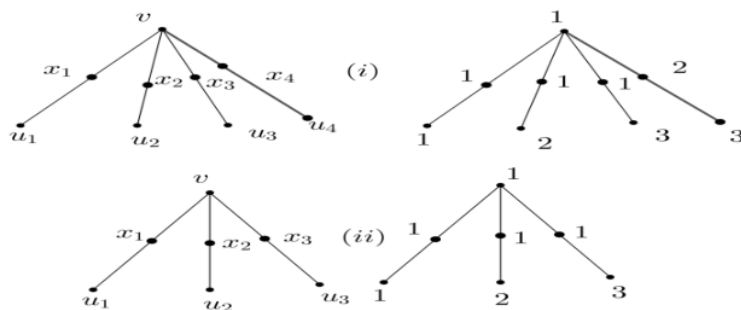


Figure 2. (i) The resolving partition of $S(K_{1,4})$ (ii) The resolving partition of $S(K_{1,3})$.

Table 1. The representation of vertices in $S(K_{1,4})$ and $S(K_{1,3})$.

Vertices	$r(v \Pi)$ in $S(K_{1,4})$	Vertices	$r(v \Pi)$ in $S(K_{1,3})$
$v \in L_1$	(0,1,2)	$v \in L_1$	(0,2,2)
$u_1 \in L_1$	(0,3,4)	$u_1 \in L_1$	(0,4,4)
$x_1 \in L_1$	(0,2,3)	$x_1 \in L_1$	(0,3,3)
$x_2 \in L_1$	(0,1,2)	$x_2 \in L_1$	(0,1,3)
$x_3 \in L_1$	(0,2,1)	$x_3 \in L_1$	(0,3,1)
$u_2 \in L_2$	(1,0,4)	$u_2 \in L_2$	(1,0,4)
$x_4 \in L_2$	(1,0,1)	$u_3 \in L_3$	(1,4,0)
$u_4 \in L_3$	(1,3,0)		
$u_3 \in L_3$	(2,1,0)		

For $n = 2$, the graph $S(K_{1,n})$ is a path, so we have $pd(S(K_{1,n})) = 2$. Base on the Lemma 3.3 and 3.4, we obtain the following Teorema 3.5

Theorem 3.5. $pd(S(K_{1,n})) = \begin{cases} 2 & \text{if } n = 2, \\ 3 & \text{if } n = 3,4, \\ p & \text{if } n \geq 5 \end{cases}$

where p is a smallest positive integer satisfying $p(p-1) + 1 \geq n$.

The Lemma 3.3 can be generalized with the subdivision operation in k_1 times on each edge e_i where $i \in [1, n]$. Let G be a connected graph, and $e_i \in E(G)$ with $i \in [1, |E(G)|]$. The subdivision graph $S(G(E; k_1, k_2, \dots, k_m))$ is a graph obtaining from graph G with any edge e_i replaced by P_{k_i+2} where $i \in [1, |E(G)|]$. On the following lemma, we give the upper bound of partition dimension of subdivision graph, $pd(S(G(E; k_1, k_2, \dots, k_m)))$ where G is a star graph $G = K_{1,n}$. On Theorem 3.6, we use the notation $x_{i,1}, x_{i,2}, \dots, x_{i,k_i}$ for subdivision vertices on edge e_i .

Theorem 3.6. Let $G = K_{1,n}$, $F = S(G(E; k_1, k_2, \dots, k_n))$, $k_i \geq 1$ and $i \in [1, n]$. For $n \geq 5$, $pd(F) \leq p$ where p is a smallest positive integer satisfying $p(p-1) + 1 \geq n$.

Proof. Let $E(G) = \{e_1, e_2, \dots, e_m\}$. The graph F can be considered as a extended of subdivision graph $S(G)$. In other words, the edge $x_i u_i$ with $i \in [1, n]$ is extended by linking u_i to new vertices in a path. Therefore, vertex u_i in $S(G)$ is considered as x_2 in graph F . Let Π' be a partition of $V(F)$ and Π be a resolving partition of $S(G)$. In graph F , the new vertices which are linked to u_i are located in partition class containing u_i .

Let Π be a resolving partition of $S(G)$ such as defining partition Π in the Lemma 3.3. Therefore, we define a new partition $\Pi' = \{L'_1, L'_2, \dots, L'_p\}$ of $V(F)$ where $L'_i = L_i \cup \{x_{i,t} | 1 \leq t \leq k_i, u_i \in L_i\}$ with $i \in [1, p]$. Let x, y be two distinct vertices in the same partition class of Π' . By Π' is a partition which is extend from Π in $S(G)$, so we suppose that $B = \{v, x_{i,1}, x_{i,2} | 1 \leq i \leq n\}$, then we consider the representation for all vertices $x \in B$. This implies that the representation for any $x \in B \subseteq V(F)$ is equal to representation of $x \in V(G) \subseteq V(F)$. So, we obtain that for two distinct vertices $x, y \in B$, $r(x|\Pi') \neq r(y|\Pi')$.

Now, we consider x or y which are not in B . For $x \in B$ and $y \notin B$, $r(y|\Pi')$ has a component which is value at least 4 but the representation $r(x|\Pi')$ has all the components which have value at most 3. So, the vertices x and y are distinguished. For $x, y \notin B$, If x, y are located on the same subdivision edge vu_i then the vertices x, y are distinguished by L_1 atau L'_t for some $t \in [1, p]$ where L_t contain the vertex $x_{i,1}$ of $S(G)$.

If both of x and y are not located on the distinct subdivision edge vu_i , for instance x is a subdivision vertex on vu_i and y is a subdivision vertex on vu_j , then the vertices x, y are distinguished by L'_t or L_r for some $t, t \in [1, p]$ where L_t contains $x_{i,1}$ and L_r contains $x_{j,1}$ of $S(G)$. These implies that Π' is a resolving partition of F . So, we have $pd(F) \leq p$.

4. Conclusion

In this paper, we have given the partition dimensions of subdivision graph on all edges in one time $S(G)$ and subdivision graph on all edges in more than one-time $S(G(E, k_1, k_2, \dots, k_n))$. On the results, we focused on the star graph $G = K_{1,n}$. The results show that the partition dimension of subdivision graph as follows:

- i. $pd(S(K_{1,n})) \begin{cases} 2 & \text{if } n = 2 \\ 3 & \text{if } n = 3, 4 \text{ where } p \text{ is a smallest positive integer satisfying } p(p-1) + 1 \geq n. \\ p & \text{if } n > 4 \end{cases}$
- ii. $pd(S(G(E; k_1, k_2, \dots, k_n))) \leq p$ where p is a smallest positive integer satisfying $p(p-1) + 1 \leq n$ and $k_i \geq 1$.

5. References

- [1] Beerliova Z, Eberhard F, Erlebach T, Hall A, Hoffmann M, Mihalak M and Ram L S 2006 Network discovery and verification. *IEEE Journal on Selected Areas in Communications* **24** 12 2168-2181.
- [2] Khuller S, Raghavachari B and Rosenfeld A 1996 Landmarks in graphs. *Discrete Applied Mathematics* **70** 3 217-229.
- [3] Caceres J, Hermendo C, Mora M, Pelayo I M, Puertas M L, Seara C and Wood D R 2007 On the Metric Dimension of Cartesian Products of Graphs. *SIAM Journal on Discrete Mathematics* **21** 2 423-441.
- [4] Chvatal V 1983, Mastermind, *Combinatorica* **3** 3-4 325-329
- [5] Yero, Ismael G, Dorota K, and Rodriguez-Velzquez J A 2011 On the metric dimension of corona product graphs. *Computers & Mathematics with Applications* **61** 9 2793-2798.
- [6] Yero, Ismael G, and Rodriguez-Velzquez J A 2010 A note on the partition dimension of Cartesian product graphs. *Applied Mathematics and Computation* **217** 7 3571-3574.
- [7] Alfarisi R, Darmaji and Dafik 2018 On the star partition dimension of comb product of cycle and complete graph. *Journal of Physics: Conference Series* 1022 012003.
- [8] Amrullah, Assiyatun H, Baskoro E T, Uttunggadewa S, Simanjuntak R 2013 The Partition Dimension for a Subdivision of Homogeneous Caterpillars. *AKCE International Journal of Graphs and Combinatorics* **10** 3, 317-325.
- [9] Amrullah, Baskoro E T, Simanjuntak R and Uttunggadewa S 2015 The Partition Dimension of a Subdivision of a Complete Graph. *Procedia Computer Science* **74** 53-59
- [10] Chartrand G, Salehi E, and Zhang P 2000 The partition dimension of graph. *Aequationes Mathematicae* **59** 45-54.
- [11] Chartrand G, Salehi E, and Zhang P 1998 On the partition dimension of graph. *Congressus Numerantium* **130** 157-168.

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