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by Amrullah Amrullah

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Citation: AIP Conference Proceedings **1707**, 020001 (2016); doi: 10.1063/1.4940802
View online: http://dx.doi.org/10.1063/1.4940802
View Table of Contents: http://aip.scitation.org/toc/apc/1707/1
Published by the American Institute of Physics

The Partition Dimension of Subdivision of a Graph

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Abstract.

Let $G=(V,E)$ be a connected graph, $u,v \in V(G)$, $e=uv \in E(G)$ and k be a positive integer. A $k-$subdivision of an edge e is a replacement of $e=uv$ with a path $u,x_1,x_2,x\cdots,x_k,v$. A graph G with a $k-$subdivided edge is denoted with $S(G(e;k))$. Let p be a positive integer and $\Pi=\{L_1,L_2,L_3,...,L_p\}$ be a *p-partition* of $V(G)$. The representation of a vertex v with respect to Π, $r(v|\Pi)$, is the vector $(d(v,L_1),d(v,L_2)\ ,d(v,L_3),...,d(v,L_p))$ where $d(v,L_i)$ for $i\in[1,p]$ is the minimum distance between v and the vertices of L_i. The partition Π is called a resolving partition of G if $r(w|\Pi)\neq r(v|\Pi)$ for all $w\neq v\in V(G)$. The partition dimension, $pd(G)$, of G is the smallest integer p such that G has a resolving p-partition. In this paper, we present sharp upper and lower bounds of the partition dimension of $S(G(e;k))$ for any graph G.

Keywords: Subdivision, partition dimension, resolving partition.
PACS: 02.10.Ox

INTRODUCTION

The concept of partition dimension of a graph was proposed by [4]. The concept was triggered by the study of metric dimension which was introduced independently by [12] and [8]. There are many papers were published in this topic of reasearch [4, 5, 10, 3, 13, 7]. The partition dimension of certain classes of graphs have appeared in many papers [1, 2, 6, 11]. However, the problems of determining the partition dimension of general connected graphs are still unresolved. For some classes of graphs, only upper or lower bounds of their partition dimension are known, the exact values are remain unknown. For instance, Juan, A. R [13] has published an upper bound for partition dimension of a tree.

Let $G=(V,E)$ be a connected graph. The *distance* $d(u,v)$ between two vertices u and v in G is the length of the shortest path between u and v. The *distance* between a vertex v and a subset L of $V(G)$ is $d(v,L)=min\{d(v,x)|x\in L\}$. Let $\Pi=\{L_1,L_2,L_3,...,L_k\}$ be a *k-partition* of $V(G)$. The *representation* of a vertex v with respect to Π, denoted by $r(v|\Pi)$, is the vector $(d(v,L_1),d(v,L_2)\ ,d(v,L_3),...,d(v,L_k))$. The partition Π is called a *resolving partition* of G if $r(w|\Pi)\neq r(v|\Pi)$ for all $w\neq v\in V(G)$. The *partition dimension* $pd(G)$ of G is the smallest integer k such that G has a resolving k-partition. If the vertices u and v are in the same partition class under Π then we write $u\sim_{\Pi} v$, otherwise $u\nsim_{\Pi} v$. Let u,v be vertices of G, if $d(u,L_i)\neq d(v,L_i)$ then we shall say that u and v are *distinguished* by L_i or u and v are *distinguishable*. Let $v\in L_i$, if $d(v,L_j)=1$ for any $j\neq i$ then v is called a *dominant vertex*. Let L_t be a partition class distinguishing two vertices u,v where $t\in[1,p]$. Vertices x and y in L_t are called the *distance defining vertices* of u and v if $d(u,L_t)=d(u,x)$, $d(v,L_t)=d(v,y)$ and $d(u,x)\neq d(v,y)$.

Let $e=uv\in E(G)$ and k be a positive integer. A $k-$subdivions of an edge e is a replacement of $e=uv$ with a path $u,x_1,x_2,x\cdots,x_k,v$. A graf G with a $k-$subdivided edge is denoted with $S(G(e;k))$. The new vertices $x_1,x_2,\cdots,x_k$ of $S(G(e;k))$ are called subdivision vertices of $S(G(e;k))$. For some graph G, we have $pd(S(G(e;k)))=pd(G)$, for instance if G is a path or a cycle. If G is a star $K_{1,4}$ then for any edge e, $pd(S(G(e;k)))\leq pd(G)$, (see Fig. 1). If G is a caterpillar $C(3;3)$ then for any pendant edge e, $pd(S(G(e;1)))\geq pd(G)$, (Fig. 2). In this paper, we present upper and lower bounds of partition dimension of $S(G(e;k))$ for arbitrary connected graph G, edge e, and positive integer k.

Proceedings of The 7th SEAMS UGM International Conference on Mathematics and Its Applications 2015
AIP Conf. Proc. 1707, 020001-1–020001-5; doi: 10.1063/1.4940802

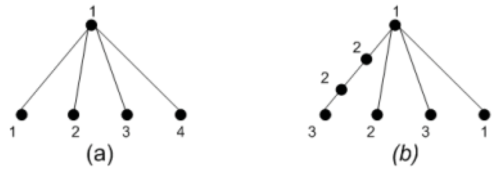

FIGURE 1. a) Graph $G = K_{1,4}, pd(G) = 4$, b) Graph $S(G(e;2))$,where e is a pendant edge, $pd(S(G(e;2))) = 3$.

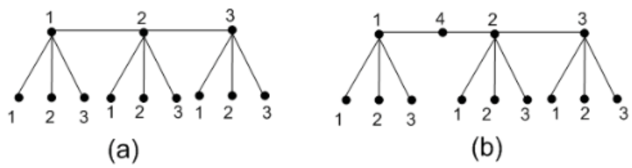

FIGURE 2. a) Graph $G = C(3;3), pd(G) = 3$, b) Graph $S(G(e;1))$,where e is a non pendant edge, $pd(S(G(e;1))) = 4$.

PREVIOUS LEMMAS

The following lemma is very useful in determining our main results.

Lemma 1. *[4] Let G be a connected non trivial graph, Π be a resolving partition of $V(G)$, and $u,v \in V(G)$. If $d(u,w) = d(v,w)$ for all $w \in V(G) - \{u,v\}$ then vertices u,v must be in distinct class partitions of* Π.

The following result is a direct consequence of Lemma 1. The result give a lower bound of partition dimension of a graph.

Corollary 1. *[4] Let G be a connected graph, if G has a vertex having k leaves then* $pd(G) \geq k$.

The following theorem shows that a graph other than a path has $pd(G) > 2$.

Theorem 1. *[5] Let G be a connected graph of order $n \geq 2$. Then $pd(G) = 2$ if only if $G = P_n$.*

MAIN RESULTS

In this section, we show that the upper bound of $pd(S(G(e;k)))$ is one more from the value of $pd(G)$. We begin with properties of distance defining vertices of two distinct vertices in the graph.

Lemma 2. *Let G be a connected graph of order $n \geq 2$, $e = ab \in E(G)$, $u,v \in V(G)$ and $\Pi = \{L_1, L_2, ..., L_p\}$ be a resolving partition of G. Let x,y be the distance defining vertices of u, v, respectively. If $d(u,a) = d(v,a)$, then both the shortest paths from u to x and from v to y do not go through e. Furthermore, if the shortest path from u to x does not go through e and the path from v to y does through e, then $d(u,x) < d(v,y)$.*

Proof. Let L_t be a partition class distinguishing u,v and containing x,y. For a contradiction, assume that both of the shortest path from u to x and from v to y are through e. Hence, we have $d(u,L_t) = d(u,a) + d(a,x)$ and $d(v,L_t) = d(v,a) + d(a,y)$. Since $x,y \in L_t$, $d(a,L_t) = min\{d(a,y), d(a,x)\}$, $d(u,L_t) = d(u,a) + d(a,L_t)$ and $d(v,L_t) = d(v,a) + d(a,L_t)$. This implies that $d(u,L_t) = d(v,L_t)$, a contradiction.

Furthermore, let assume that a shortest path from u to x is not through e and the shortest path from v to y is through e. Since $d(u,L_t) \neq d(v,L_t)$, this means that $d(u,L_t) > d(v,L_t)$ or $d(u,L_t) < d(v,L_t)$. Therefore, $d(u,x) > d(v,y)$ or $d(u,x) < d(v,y)$. Now, we assume that $d(u,x) > d(v,y)$. We know that $d(u,x) > d(v,a) + d(a,b) + d(b,y)$. Since $d(u,a) = d(v,a)$, we obtain $d(u,x) > d(u,a) + d(a,b) + d(b,y)$. We have $d(u,x) > d(u,y)$, a contradiction, and we obtain $d(u,x) < d(v,y)$. □

Lemma 3. *Let G be a connected graph of order $n \geq 2$, $u,v \in V(G)$, $e = ab \in E(G)$ and $\Pi = \{L_1, L_2, ..., L_p\}$ be a resolving partition of G. Let x,y be two distance defining vertices of u,v. If $d(u,a) = d(v,b)$, then both the shortest path form u to x and from v to y are not through e.*
Furthermore, if a shortest path from u to x is not through e and a shortest path from v to y is through e, then $d(u,x) < d(v,y)$).

Proof. Let L_t be a partition class distinguishing u,v and containing x,y. For a contradiction, we assume that both of the shortest path form u to x and v to y are through the edge e. Let Q_1 be a shortest path from u to x passing through $e = ab$ and Q_2 be a shortest path from v to y passing through $e = ab$. Let $d(v,y) = r_2$ and $d(u,x) = r_1$. We suppose $r_1 < r_2$. Since $d(u,a) = d(v,b)$ and $r_1 < r_2$, we obtain $d(b,x) < d(a,y)$. So, we have $d(b,L_t) < d(a,L_t)$. Therefore, we obtain $d(v,L_t) = d(v,b) + d(b,L_t) < d(v,b) + d(b,a) + d(a,y)$. This implies that y is not a distance defining vertex of v, a contradiction.

Furthermore, for the shortest path from u to x is not through e and the shortest path from v to y is through e. Since $d(u,L_t) \neq d(v,L_t)$, this means that $d(u,L_t) > d(v,L_t)$ or $d(u,L_t) < d(v,L_t)$. Therefore, $d(u,x) > d(v,y)$ or $d(u,x) < d(v,y)$.

we assume that $d(u,x) > d(v,y)$. So we have $d(u,x) > d(v,b) + d(b,a) + d(a,y)$. Since $d(u,a) = d(v,b)$, we obtain $d(u,x) > d(u,a) + d(b,a) + d(a,y)$. Thereore, we have $d(u,x) > d(u,a) + d(a,y)$, we get $d(u,x) > d(u,y)$. This implies that x is not a distance defining vertex of u, a contradiction. As consequence, we have $d(u,x) < d(v,y)$. □

The following Theorem 2 give an upper bound for partition dimension of $S(G(e;k))$.

Theorem 2. *Let G be a connected graph of order $n \geq 2$ and $e \in E(G)$. Then $pd(S(G(e;k))) \leq pd(G) + 1$ and the bound is best possible.*

Proof. Let $e = ab \in E(G)$. If G is a path, then clearly $S(G(e;k))$ is a path too. So, we obtain $pd(S(G(e;k)) = 2 = pd(G)$. If G is not a path, and by Theorem 1, then we have $pd(G) = p \geq 3$. Let $\Pi = \{L_1, L_2, \cdots, L_p\}$ be a resolving partition of G where $p \geq 3$. Let $\Pi' = \{L'_1, L'_2, ..., L'_p, L'_{p+1}\}$ be a partition of $V(S(G(e;k)))$ where $L'_{p+1} = \{a_1, ..., a_k\}$ and $L'_i = L_i$ for $i \in [1,p]$. We will show that Π is a resolving partition of $S(G(e;k))$.

Let $u,v \in V(S(G(e;k))$ in the same partition class of Π'. By definition Π', there are not a subdivision vertex u and a non subdivision vertex v in $S(G(e;k))$. We consider u,v in two cases.

First, both of u,v are not two subdivision vertices. Suppose x,y in L_t are the distance defining vertices of u,v for some $t \in [1,p]$. In graph G, let Q_1 be a shortest path from u to x with length r_1 and Q_2 be a shortest path from v to y with length r_2. In graph $S(G(e;k))$, if $d(u,L_{p+1}) \neq d(v,L_{p+1})$, then they are distinguished by L'_{p+1}. If $d(u,L_{p+1}) = d(v,L_{p+1})$, then $d(u,a) = d(v,a)$ or $d(u,a) = d(v,b)$. By Lemma 2 and 3, both of Q_1 and Q_2 are not through e. If both of Q_1 and Q_2 do not go through e, then u,v are distinguished by L'_t. If one of Q_1 or Q_2 does not go through e, then the path no going through e has length least. Let Q_1 does not go through e and Q_2 goes through e. So, by Lemma 2, we have $r_1 < r_2$. Since e is a subdivision edge, $d(v,y)$ in $S(G(e;k))$ great than $d(v,y) = r_2$ in G. This implies $d(v,L'_t) > d(v,L_t)$ and $d(u,L'_t) = d(u,L_t)$. Therefore, we have $d(u,L'_t) > d(v,L't)$. So, u,v are distinguished by L'_t.

Second, both of u,v are two subdivision vertices. It means that $u,v \in L'_{p+1}$. If a,b in the same partitions class of Π, then u,v are distinguished by partitions class which distinguishing a or b. If a,b are in the distinct partition class of Π, then u,v are distinguished by a or b.

This upper bound of partition dimension of $S(G(e;k))$ is a sharp value, because there is an example where $pd(S(G(e;1))) = 4 = pd(G)$ (see Fig. 2). □

The following theorem give the lower bound of partition dimension of $S(G(e;k))$ where e is a pendant edge.

Theorem 3. *Let G be a connected graph and $e \in E(G)$. If e is a pendant edge, then $pd(S(G(e;k))) \geq pd(G) - 1$.*

Proof. For a contradiction, assume that $\Pi = \{L_1, L_2, \cdots, L_{p-2}\}$ is a resolving partition of $S(G(e;k))$. Let $e = uv \in E(G)$ be a pendant edge where v is a leaf, and $x_1, x_2, \cdots, x_k$ are subdivision vertices in $S(G(e;k))$. Suppose $v \in L_s$ and $u \in L_t$ for some $s,t \in [1, p-2]$. On $S(G(e;k))$, the path $u, x_1, x_2, \cdots, x_k, v$ is replaced by an edge uv, so we obtain a graph $G' = G$. Define a new partition $\Pi' = \{L'_1, L'_2, \cdots, L'_{p-2}, L'_{p-1}\}$ of $V(G')$ where $L'_i = L_i$ for $i \notin \{s, p-1\}$, $L'_s = L_s - \{v\}$ and $L'_{p-1} = \{v\}$. We will show that Π' is a resolving partition of G'.

Let w,z be two distinct vertices in the same partition class of Π'. Let x,y be two distance defining vertices of w,z, respectively. Suppose $B = \{x_1, x_2, ..., x_k, v\}$. If $d(w,v) \neq d(z,v)$, then w,z are distinguished by L'_{p-1}. If $d(w,v) = d(z,v)$, then there is at most one of $\{x,y\}$ belongs to B. If both of $x,y \notin B$, then w,z are distinguished by $L'_t = L_t$ of Π', because subdivision on e does not impact on $d(u,x)$ and $d(v,y)$. If one of x,y belongs to B, suppose $x \in B$, then we have

$d(z,y) < d(w,x)$ in $S(G(e;k))$. Since the path $u, x_1, x_2, \cdots, x_k, v$ is replaced by $e = uv$ and $d(w,x') \geq d(w,x)$ for all $x' \in L_t$, then $d(w, L'_t) \geq d(w, L_t)$. On the other side, the shortest path from z to y is not impacted by subdivision on e, then $d(z, L'_t) = d(z, L_t)$. This implies, w, z in G' are distinguished by L'_t of Π'. As a consequence, Π' is a resolving partition of $G' = G$, a contradiction. □

Referring to Theorems 3, 2 and 1, we obtain an upper bound and the lower bound of partition dimension of subdivision graph on a pendant edge as the following.

Corollary 2. *Let G be a connected graph and $pd(G) = p$. If e is a pendant edge, then*

1. $pd(S(G(e;k))) = p$ *if* $p = 2$,
2. $p \leq pd(S(G(e;k))) \leq p+1$ *if* $p = 3$, .
3. $p-1 \leq pd(S(G(e;k))) \leq p+1$ *if* $p \geq 4$.

In the next lemma, we give a lower bound for partition dimension of a graph G where G has three distinct vertices which are each adjacent to three leaves.

Theorem 4. *Let G be a connected graph with order $n \geq 13$. If G has three distinct vertices x_1, x_2, x_3 where each x_i is adjacent to three leaves, then $pd(G) \geq 4$.*

Proof. For a contradiction, assume that $\Pi = \{L_1, L_2, L_3\}$ is a resolving partition of G. We suppose that x_i is adjacent to three leaves $w_{i,1}, w_{i,2}, w_{i,3}$, for $i \in [1,3]$. Since each of x_1, x_2, x_3 is adjacent to 3 leaves which are in the distinct partition classes of Π, then x_1, x_2, x_3 are in distinct partition classes of Π. Let $x_1 \in L_1$, $x_2 \in L_2$, $x_3 \in L_3$ and $w_{i,j} \in L_j$, for $j \in [1,3]$.
Since $n \geq 13$, there exits a vertex $u \in V(G)$ such that $ux_i \in E(G)$ and $u \notin \{x_1, x_2, x_3\} \cup \{w_{i,j} | 1 \leq i, j \leq 3\}$. Let $ux_2 \in E(G)$. Now, consider u in the partition class of Π.
If $u \in L_1$ then we obtain that $r(u|\Pi) \in \{(0,1,2), (0,1,1)\}$. If $r(u|\Pi) = (0,1,2)$, then we have $r(u|\Pi) = r(w_{2,1}|\Pi)$. If $r(u|\Pi) = (0,1,1)$, then we have $r(u|\Pi) = r(v_2|\Pi)$. So, this implies that $u \notin L_1$.
If $u \in L_2$ then we obtain that $r(u|\Pi) \in \{(2,0,2), (2,0,1), (1,0,2), (1,0,1)\}$. It is a contradiction, since all possibilities of representation of u are used by $v_2, w_{1,2}, w_{2,2}, w_{3,2}$. So, this implies that $x_2 \notin L_2$.
If $u \in L_3$ then we obtain that $r(u|\Pi) \in \{(2,1,0), (1,1,0)\}$. It is also a contradiction, since these representations are used by $v_3, w_{2,3}$. So, this implies that $u \notin L_3$.
Therefore that $u \notin L_1 \cup L_2 \cup L_3$. So, we have that $pd((S(G(e;k)) \geq 4$. □

A consequence of Theorem 4 gives an lower bound for partition dimension of $S(G(e;k))$ where G of order $n \geq 13$ has three distinct vertices which is adjacent to three leaves.

Corollary 3. *Let G be a connected graf of $n \geq 12$. If G has three distinct vertices x_1, x_2, x_3 where each x_i is adjacent to three leaves, then $pd(S(G(e;k))) \geq 4$ for any non pendant edge e.*

Let G be a connected graph and $u \in V(G)$. Let $K_{1,r}$ be a star with center v and pendants $\{w_1, w_2, \cdots, w_r\}$. We define $G*$ as a graph with $V(G*) = V(G) \cup V(K_{1,r})$ and $E(G*) = E(G) \cup E(K_{1,r}) \cup \{uw_r\}$. Next lemma gives a property of $G*$.

Lemma 4. *If Π is an $r-1$ resolving partition of $G*$, then $u \nsim_{\Pi} v$ and $u \nsim_{\Pi} w_j$.*

Proof. Let $\Pi = \{L_1, L_2, ..., L_{r-1}\}$ be a resolving partition of $G*$. By definition $G*$, we have w_j is adjacent to u, v and $deg(w_j) = 2$. Therefore, v is adjacent to $r-1$ leaves $w_1, w_2, \cdots, w_{r-1}$. So, by Lemma 1, they are in distinct partition classes. Let $w'_i \in L_i$, for all $i \in [1, r-1]$. So, we have that v is a dominant vertex.

For a contradiction, assume that $u \sim_{\Pi} v$. Let $u, v \in L_1$. Since $u \sim_{\Pi} v$ and $d(w_j, L_i) \leq 2$, we obtain than $r(w'_t|\Pi) = r(w_j|\Pi)$, for some $t \in [1, r-1]$, a contradiction.

Now, assume that $w_j \sim_{\Pi} u$. Let $w_j, u \in L_1$. Since v is a dominant vertex and Π is $r-1$-resolving partition, then w_j is in the same partition class with some w'_t for some $t \in [1, r-1]$. So, since $d(w_j, L_i) \leq 2$, we obtain $r(w'_t|\Pi) = r(w_j|\Pi)$, a contradiction. Therefore we have $w_j \nsim_{\Pi} u$. □

Theorem 5. *Let G be a connected graph, $v_1, v_2 \in V(G)$, and $e = v_1v_2 \in E(G)$. Let v_i be adjacent to 3 leaves, for each $i \in [1,2]$. Then $pd(S(G(e;4))) \geq 4$.*

Proof. Let $w_{i,1}, w_{i,2}, w_{i,3}$ be leaves which are adjacent to v_i for each $i \in [1,2]$. By a contradiction, assume that $\Pi = \{L_1, L_2, L_3\}$ is a resolving partition of $S(G(e;4))$. Let $v_1 \in L_1$ and $v_2 \in L_2$. Since Π is 3-resolving partition, $w_{i,j} \in L_j$ for all $i \in [1,2]$ and $j \in [1,3]$. By Lemma 4, we have $x_2 \in L_2$ or $x_2 \in L_3$.

Case 1: $x_2 \in L_2$.

By Lemma 4, we have $x_1 \in L_3$. Since $v_2 \in L_2$, and by Lemma 4, we obtain $x_3 \in L_1$ or $x_3 \in L_3$. If $x_3 \in L_1$, we obtain $r(v_2|\Pi) = r(x_2|\Pi)$. So, we have $x_3 \in L_3$. By Lemma 4, x_4 must be in L_1 or L_2. If $x_4 \in L_1$, then $r(x_4|\Pi) = r(v_1|\Pi)$. If $x_4 \in L_2$, then $r(x_4|\Pi') = r(x_2|\Pi)$. This implies $x_2 \notin L_2$.

Case 2: $x_2 \in L_3$.

By Lemma 4, we have $x_1 \in L_2$ or $x_1 \in L_1$. If $x_1 \in L_2$ then we obtain $r(x_1|\Pi) = r(v_2|\Pi)$, so $x_1 \notin L_2$. If $x_1 \in L_1$, then consider x_3 and x_4. Since $v_2 \in L_2$, and by Lemma 4, we obtain $x_3 \notin L_2$ and $x_3 \nsim_{\Pi} x_4$. Now, we have four cases of x_3 and x_4 which satisfy the conditions, namely (a) $x_3 \in L_1$ and $x_4 \in L_2$,. This condition implies $r(x_3|\Pi) = r(v_1|\Pi)$. (b) $x_3 \in L_1$ and $x_4 \in L_3$. This implies $r(x_1|\Pi) = r(x_3|\Pi)$. (c) $x_3 \in L_3$ and $x_4 \in L_1$. This implies $r(x_3|\Pi) = r(v_{13}|\Pi)$. (d) $x_3 \in L_3$ and $x_4 \in L_2$. The last condition implies $r(x_3|\Pi) = r(v_{23}|\Pi)$. So, base on four conditions, we obtain $x_2 \notin L_3$. Referring to cases 1 and 2, we have $x_2 \notin L_1 \cup L_2 \cup L_3$. So, we obtain $pd(S(G(e;k)) \geq 4$. □

ACKNOWLEDGMENTS

This research was supported Research Grant "Hibah Riset Unggulan ITB-DIKTI", Indonesian Ministry of Research, Technology and Higher Education

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