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*by Adhitya Wisnu*

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# 1 Coprime Graph of Integer Modulo $n$ Group and its Subgroups

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**Abstract.** Coprime Graph is a geometric representation of a group in the form of undirected graph. The coprime graph of a group  $G$ , denoted by  $\Gamma_G$  is a graph whose vertices are all elements of group  $G$ ; and two distinct vertices  $a$  and  $b$  are adjacent if and only if  $(|a|; |b|) = 1$ . In this paper, we study coprime graph of integers modulo  $n$  group and its subgroups. One of the results is if  $n$  is a prime number, then coprime graph of integers modulo  $n$  group is a bipartite graph.

**Keywords:** bipartite graph, coprime graph, integer modulo, multipartite graph.

## I. INTRODUCTION

Mathematicians define specific graphs on algebraic structures, and use graph properties as a geometric representations of an algebraic structure. In 2014, Ma *et al* [1] define a coprime graph of a group as follows: take  $G$  as the vertices of  $\Gamma_G$  and two distinct vertices  $x$  and  $y$  are adjacent if and only if  $(|x|, |y|) = 1$ . In this paper, we will study the coprime graph of cyclic group,  $\mathbb{Z}_n$ . In 2016 Dorbidi [2] classify all the groups which  $\Gamma_G$  is a complete  $r$ -partite graph or a planar graph, he also studied the automorphism group of  $\Gamma_G$ .

## II. Result

### 2.1. Coprime Graph of $\mathbb{Z}_n$

Some terminology of group and graph that used in this paper are given as follows.

**Definition 1** ([3]) Two vertices on the non-directed graph  $G$  are said to be neighbors if they are connected directly by an edge. In other words,  $u$  is adjacent to  $v$  if  $(u, v)$  is an edge on graph  $G$ .

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**Definition 2** If  $G$  is a group with identity  $e$  and  $x \in G$ , the order of  $x$  is the least natural number  $k$  such that  $x^k = e$  and we write  $|x| = k$ .

1  
**Definition 3** ([1]) The coprime graph of a group  $G$ , denoted by  $\Gamma_G$  is a graph whose vertices are elements of  $G$  and two distinct vertices  $u$  and  $v$  are adjacent if and only if  $(|u|, |v|) = 1$ .

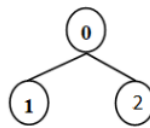
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**Definition 4** ([3]) Graph  $G$ , whose set of vertices can be partitioned into two subsets  $V_1$  and  $V_2$ , such that each edge in  $G$  connecting a vertex in  $V_1$  to a vertex in  $V_2$ , is called a bipartite graph

and is expressed as  $G(V_1, V_2)$ . In other words, each pair of vertices in  $V_1$  (as well as vertices in  $V_2$ ) are not neighbors. If each node in  $V_1$  is adjacent to all vertices at  $V_2$ , then  $G(V_1, V_2)$  is called a complete bipartite graph, denoted by  $K(m, n)$ , where  $m = |V_1|$  and  $n = |V_2|$ .

**Definition 5** ([1]) A  $k$ -partite graph is a graph whose vertices can be partitioned into  $k$  disjoint sets so that no two vertices within the same set are adjacent.

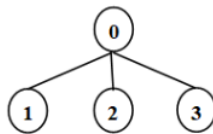
As we know,  $\mathbb{Z}_n$  is a cyclic group. The elements of  $\mathbb{Z}_n$  can be written as  $\mathbb{Z}_n = \{0, 1, 2, \dots, n-1\}$ . Some examples of coprime graphs that obtained from the group  $\mathbb{Z}_n$  are as follow.

**Example 1** Let  $\mathbb{Z}_3 = \{0, 1, 2\}$ . We can see that the order of its elements are  $|0| = 1, |1| = 3, |2| = 3$ . Therefore, we have the coprime graph of  $\mathbb{Z}_3$  as shown in Figure 1..



**Figure 1.** Coprime graph of  $\mathbb{Z}_3$

**Example 2** Let  $\mathbb{Z}_4 = \{0, 1, 2, 3\}$ . We can check that the order of its elements are  $|0| = 1, |1| = 4, |2| = 2, |3| = 4$ . Therefore, we have the coprime graph of  $\mathbb{Z}_4$  as shown in Figure 2..



**Figure 2.** Coprime graph of  $\mathbb{Z}_4$

By following the above examples, we can obtain some properties of the coprime graph of Group  $\mathbb{Z}_n$  as follow. The first results we obtained is the coprime graph of  $\mathbb{Z}_n$  is a complete bipartite graph whenever  $n$  is a prime.

**Theorem 1** If  $n$  is a prime number, then the coprime graph of  $\mathbb{Z}_n$  is a complete bipartite graph.

*Proof.* Clearly  $\mathbb{Z}_n = \{0, 1, 2, \dots, n-1\}$  with  $|0| = 1$ . Since  $n$  is a prime number, then  $|1| = |2| = \dots = |n-1| = n$ . So, the set  $\mathbb{Z}_n = \{0, 1, 2, \dots, n-1\}$  can be partitioned into  $V_1 = \{0\}$  and  $V_2 = \{1, 2, \dots, n-1\}$ . For all  $a, b \in V_2$ , we have  $(|a|, |b|) = n > 1$ . This implies  $a$  and  $b$  are not neighbors. Because  $|0| = 1$ , then for each  $a \in V_2$ , we have  $(|0|, |a|) = 1$ . So 0 is adjacent to  $a$ . Thus coprime graph of the group  $\mathbb{Z}_n$  is a complete bipartite graph.  $\square$

The second results we obtained is the coprime graph of  $\mathbb{Z}_n$  is a complete bipartite graph whenever  $n$  is a prime power.

**Theorem 2** If  $n = p^k$ , for some prime  $p$  and  $k \in \mathbb{N}$ , then the coprime graph of  $\mathbb{Z}_n$  is a complete bipartite graph.

*Proof.* Clearly  $\mathbb{Z}_n = \{0, 1, 2, \dots, p^{k-1}\}$  with  $|0| = 1$ . Since  $p$  is a prime number, every  $a \in \mathbb{Z}_n$  with  $(p^k, a) \neq 1$ , can be written as  $a = p^l q$ , for some  $l$  with  $l < k$ . This implies  $|a| = p^{k-l}$ . Also, for every  $b \in \mathbb{Z}_n$  with  $(p^k, b) = 1$ , we have  $|b| = p^k$ . So, for every  $a, b \in \mathbb{Z}_n$  with  $a, b \neq 0$ , we have  $(|a|, |b|) \neq 1$ . Thus,  $\mathbb{Z}_n = \{0, 1, 2, \dots, p^{k-1}\}$  can be partitioned into  $V_1 = \{0\}$  and  $V_2 = \{1, 2, \dots, p^{k-1}\}$ . Because  $|0| = 1$ , then for each  $a \in V_2$ , we have  $(|a|, |0|) = 1$ . Then, for all  $a \in V_2$ ,  $a$  is adjacent to 0, thus coprime graph which is formed from  $\mathbb{Z}_n$  is a complete bipartite graph.  $\square$

The second results we obtained is the coprime graph of  $\mathbb{Z}_n$  is a  $t$ -partite graph whenever  $n$  is not a prime power.

**Theorem 3** If  $n = p_1^{k_1} p_2^{k_2} \cdots p_j^{k_j}$ , where  $p_1, p_2, \dots, p_j$  are distinct prime numbers and  $k_1, k_2, \dots, k_j$  are natural numbers, then coprime graph of  $\mathbb{Z}_n$  is a  $(j + 1)$ -partite graph.

*Proof.* Let  $\mathbb{Z}_n$  be the group of integers modulo  $n$ , with  $n = p_1^{k_1} p_2^{k_2} \cdots p_j^{k_j}$ , where  $p_1, p_2, \dots, p_j$  are distinct prime numbers and  $k_1, k_2, \dots, k_j \in \mathbb{N}$ . Clearly  $\mathbb{Z}_n = \{0, 1, 2, \dots, (p_1^{k_1} p_2^{k_2} \cdots p_j^{k_j}) - 1\}$ . Every  $a \in \mathbb{Z}_n$  with  $(a, n) \neq 1$ , can be written as  $a = p_1^{l_1} p_2^{l_2} \cdots p_j^{l_j}$  with  $l_i \leq k_i$ . This implies,  $|a| = (p_1^{k_1-l_1} p_2^{k_2-l_2} \cdots p_j^{k_j-l_j})$ . Any  $b \in \mathbb{Z}_n$  with  $(b, n) = 1$ , we have  $|b| = p_1^{k_1} p_2^{k_2} \cdots p_j^{k_j}$ . So,  $\mathbb{Z}_n = \{0, 1, 2, \dots, (p_1^{k_1} p_2^{k_2} \cdots p_j^{k_j}) - 1\}$  can be partitioned into the following sets.

$$V_1 = \{0\}$$

$$V_2 = \{a_1, a_2, \dots, a_j\} \text{ with } |a_i| = \prod_{w=1}^j p_w^{\alpha_w}, 0 \leq \alpha_w \leq k_w, \alpha_1 \neq 0$$

$$V_3 = \{b_1, b_2, \dots, b_j\} \text{ with } |b_i| = \prod_{w=2}^j p_w^{\alpha_w}, 0 \leq \alpha_w \leq k_w, \alpha_2 \neq 0$$

$$\vdots$$

$$V_{j+1} = \{q_1, q_2, \dots, q_j\} \text{ with } |q_i| = p_j^{\alpha_j}, 0 \leq \alpha_j \leq k_j$$

So, 0 is adjacent to all  $x \in V_i, i = 2, 3, \dots, j + 1$ . Also, some  $u \in V_i$  is adjacent to  $v \in V_l, i \neq l$ . Thus, coprime graph that formed from  $\mathbb{Z}_n$  is a graph  $(j + 1)$ -partite.  $\square$

## 2.2. Coprime Graph of Subgroups of $\mathbb{Z}_n$

In this part, we will describe coprime graphs of subgroups of  $\mathbb{Z}_n$ . The first result is the coprime graphs of nontrivial subgroups of  $\mathbb{Z}_n$  are bipartite whenever  $n$  is a prime power.

**Theorem 4** If  $n = p^k$ , for some prime number  $p$  and  $k \in \mathbb{N}$ , then coprime graphs of nontrivial subgroups of  $\mathbb{Z}_n$  are bipartite.



*Proof.* Any non-trivial subgroup of  $\mathbb{Z}_{p^k}$  is isomorphic to  $\mathbb{Z}_{p^l}$ , for some  $0 < l < k$ . Therefore, by Theorem 2, coprime graph of any nontrivial subgroup of  $\mathbb{Z}_{p^k}$  is bipartite.  $\square$

The second result is whenever  $n$  is a product of two prime power, the the coprime graphs of nontrivial subgroups of  $\mathbb{Z}_n$  are bipartite or tripartite.

**Theorem 5** If  $n = p_1^{k_1} p_2^{k_2}$ , with  $p_1, p_2$  are distinct prime numbers, and  $k_1, k_2$  are natural numbers, then coprime graphs of nontrivial subgroups of  $\mathbb{Z}_n$  are bipartite or multipartite (3-partite).

*Proof.* Any non-trivial subgroup of  $\mathbb{Z}_{p_1^{k_1} p_2^{k_2}}$  is isomorphic to  $\mathbb{Z}_{p_1^{l_1} p_2^{l_2}}$ , for some  $l_1 < k_1$  and  $l_2 < k_2$ . When  $l_1 = 0$  or  $l_2 = 0$ , then by Theorem 2, the coprime graph of the corresponding subgroup is bipartite. Otherwise, by Theorem 3, the coprime graph of the corresponding subgroup is 3-partite.  $\square$

The third result is whenever  $n$  is not a prime power, the coprime graphs of nontrivial subgroups of  $\mathbb{Z}_n$  are multipartite.

**Theorem 6** If  $n = p_1^{k_1} p_2^{k_2} \cdots p_j^{k_j}$ , where  $p_1, p_2, \dots, p_j$  are distinct prime numbers and  $k_1, k_2, \dots, k_j \in \mathbb{N}$ , then the coprime graph of non-trivial subgroups of  $\mathbb{Z}_n$  is multipartite.

*Proof.* Any non-trivial subgroup of  $\mathbb{Z}_{p_1^{k_1} \cdots p_j^{k_j}}$  is isomorphic to  $\mathbb{Z}_{p_1^{l_1} \cdots p_j^{l_j}}$ , for some  $l_i < k_i$ , for all  $i = 1, 2, \dots, j$ . If  $l_{i_1}, l_{i_2}, \dots, l_{i_t}$  are the only non-zero powers, then by Theorem 3, the coprime graph of the corresponding subgroup is  $(t + 1)$ -partite.  $\square$

### III. CONCLUSIONS

We described coprime graphs of  $\mathbb{Z}_n$  and its subgroups for all  $n$ . In general, the resulting coprime graphs are bipartite whenever  $n$  is a prime power and multipartite whenever  $n$  is not a prime power. But when we consider its subgroups, the coprime graph subgroup of  $\mathbb{Z}_n$  may a bipartite even if  $n$  is not a prime power.

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### REFERENCES

- [1] X.L. Ma, H.Q. Wei, and L.Y. Yang, "The coprime graph of a group," *International Journal of Group Theory*, vol. 3, no. 3, pp. 13–23, 2014.
- [2] H.M. Dorbidi, "A note on the coprime graph of a group," *International Journal of Group Theory*, vol. 5, no. 4, pp. 17–22, 2016.
- [3] R. Munir, *Matematika Diskrit*. Bandung: Penerbit Informatika, 2010.



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