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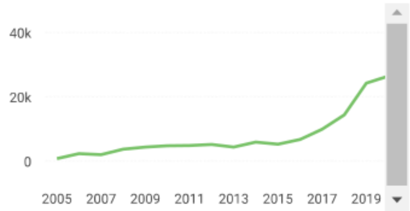

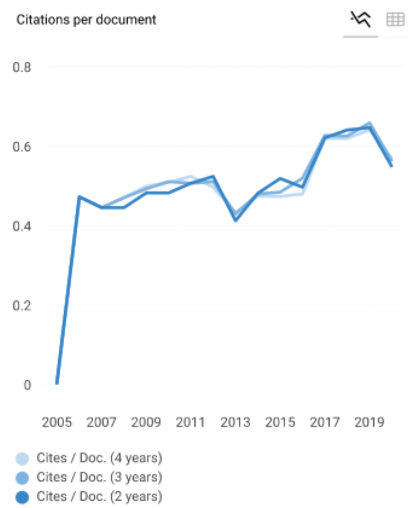

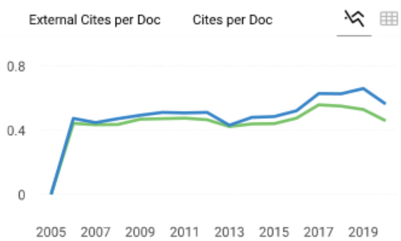

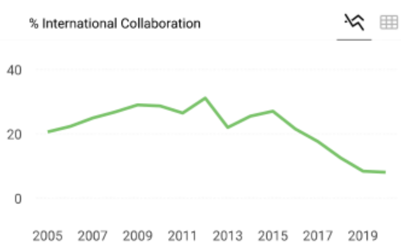

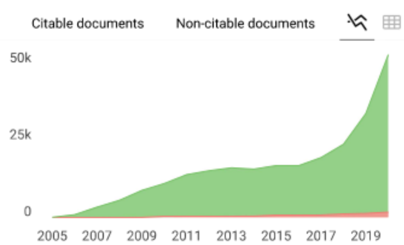

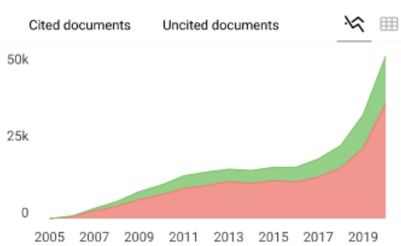

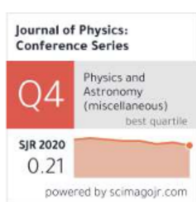

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Some characterizatsion of coprime graph of dihedral group D_{2n}

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Some characterizatsion of coprime graph of dihedral group D_{2n}

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Abstract. The Coprime graph of group G denoted Γ_G is a graph with vertices is an element of G, and two distinct vertices are adjacent when its order relative prime. In 2020, Gazir et al. give some characterizations of $\Gamma_{D_{2n}}$ for n a prime power. The method that uses in this paper is deductive proof by taking some example of a coprime graph of D_{2n}, then generalized the characterization of example. This paper gives some characteristics of the coprime graph of a dihedral group for more general cases. One of the result, $\Gamma_{D_{2n}}$ is a multipartite graph with girth 3, radius 1, and diameter 2.

1. Introduction

In recent years, mathematicians construct a graph from a mathematical system such as commuting and non-commuting graph, cycle graph, identity graph, and zero divisor graph. In 2014, X. Ma et al. defined a new graph called the coprime graph. The Coprime graph of finite group G denoted by Γ_G is a graph with vertices are elements of G and two distinct vertices x and y are adjacent if and only if $(ord(x), ord(y)) = 1$[1]. In 2017, Abdussakir researched the commuting and non-commuting graph of a dihedral group and got some characteristics of that graph such as radius, diameter, cycle multiplicity, and matrix dimensions [2]. In 2020, Gazir S. et al. found the form of the coprime graph of a dihedral group that is complete tripartite when n odd prime number and if $n = 2^k$ then $\Gamma_{D_{2n}}$ is a complete bipartite graph [3].

From the above description, we will determine the characteristics of the coprime graph of a dihedral group with n odd composite numbers, such as form, girth, radius, and diameter.

2. Method

This research's methods are deductive proof that makes conjectures based on properties and then proves them with rigorous proof. The first step is studying definitions and theories about the coprime graph of the dihedral group, then studying examples of characterizations of the coprime graph of the dihedral group. The last step is to make conjectures and prove them.

3. Result and Discussion

This section will discuss about the dihedral group and its representation in the coprime graph and some characteristics.

3.1 Coprime Graph of D_{2n}

Definition 1 ([4]) Group G, named dihedral group with order $2n$, $n \geq 3$ and $n \in \mathbb{N}$, is a group generated by $a, b \in G$ with properties

$$G = \langle a, b | a^n = e, b^2 = e, bab^{-1} = a^{-1} \rangle.$$

A dihedral group with order $2n$ denoted by D_{2n}.

Definition 2 ([5]) If $(G,*)$ any group. Let a any elements of G. The least positive integer m with $a^m = e$ (e identity in G) then m named order of a, and denoted by $|a| = m$ or $ord(a) = m$.

Definition 3 ([1]) Let G finite group, the coprime graph of G denoted by Γ_{G} is a graph with vertices are elements of G and two distinct vertices x and y are adjacent if and only if $(ord(x), ord(y)) = 1$.

These are three theorems given by Gazir S. et al. about the form of the coprime graph. The first result about the form of the coprime graph of D_{2n}, with n odd prime number explained in the next theorem.

Theorem 1 ([6]) Let If n is an odd prime number, then the coprime graph of D_{2n} is complete tripartite.

In addition, the coprime graph of D_{2n} is a complete bipartite graph when $n = 2^k$, for some $k \in N$.

Theorem 2 ([6]) Let $n = 2^k$, for some $k \in N$ then the coprime graph of D_{2n} is a complete bipartite.

The next theorem explains that the coprime graph of D_{2n} is complete tripartite for some $n = p^k$.

Theorem 3 ([6]) Let $n = p^k$ for some $k \in N$ and p is a prime number, $p \neq 2$, then the coprime graph of D_{2n} is complete tripartite.

The last theorem about the form of the coprime group of D_{2n} where n that the following Theorem gives more generalize.

Theorem 4 Let $n = p_1^{k_1} p_2^{k_2} p_3^{k_3} \dots p_m^{k_m}$ where $1 \leq i \leq m, p_i$ are distinct prime number, and $p_i \neq 2$ then the coprime graph of D_{2n} is $(m+2)$-partite.

Proof. Let D_{2n} a dihedral group with $n = p_1^{k_1} p_2^{k_2} p_3^{k_3} \dots p_m^{k_m}$ where $1 \leq i \leq m$, p_i are distinct prime number, $p_i \neq 2$. We define some set, the first set is a set of elements with order 1, the second set is a set of elements with order 2, or even the third set is a set of an element with order $p_1$and odd, and the $(m + 2)$ set is a set of elements with order p_m and odd and p_j not divide p_m where $1 \leq j \leq m - 1$, clearly these sets are a partition of D_{2n}. Let $x, y \in V_i$, thus $p_i | ord(x)$ and $p_i | ord(y)$, so $(ord(x), ord(y)) \neq 1$, then x and y are not adjacent. So, the coprime graph of D_{2n} is $m + 2$-partite. ■

3.2 Radius and Diameter

Definition 4 ([7]) Let u and v are vertices in G, the $d(u, v)$ denotes the length of the shortest path between u and v. The least distance between all pairs of the vertices of G is called the diameter of G, and is denoted by $rad(G)$.

Definition 5 ([7]) Let u and v are vertices in G, the $d(u, v)$ denotes the length of the shortest path between u and v. The largest distance between all pairs of the vertices of G is called the diameter of G, and is denoted by $diam(G)$.

The next theorem explains other characteristics of the coprime graph of the dihedral group, like radius and diameter. For any $n, rad(\Gamma_{D_{2n}}) = 1$ and $diam(\Gamma_{D_{2n}}) = 2$.

Theorem 5 Let $\Gamma_{D_{2n}}$coprime graph then $rad(\Gamma_{D_{2n}}) = 1$ and $diam(\Gamma_{D_{2n}}) = 2$.
Proof. We know that $(ord(x), ord(y)) = 1$, for each $x \in D_{2n}$. Thus, e are adjacent with all vertices in D_{2n} then $e(e) = 1$, cause the radius D_{2n} is the least eccentric in D_{2n} we get $rad(D_{2n}) = 1$. Next, let a and b two distinct vertices in D_{2n}. If a and b are adjacent, then $d(a, b) = 1$. In other, if a and b are not adjacent, then $d(a, b) = 2$. Such distances of two distinct vertices in D_{2n} is 1 or 2. Thus diameter is the largest eccentric then $diam(\Gamma_{D_{2n}}) = 2$. ∎

3.3 Girth
Definition 6 ([8]) The girth of graph G is the length of the shortest cycle contained in G.

Theorem 6 Let D_{2n} dihedral group. If $n = {p_1}^{k_1}{p_2}^{k_2} \dots {p_m}^{k_m}$ and $n \nmid 2$, then girth of $\Gamma_{D_{2n}}$ is 3.
Proof. Let D_{2n} dihedral group. If $n = {p_1}^{k_1}{p_2}^{k_2} \dots {p_m}^{k_m}$ and $2 \nmid n$. Based on Theorem 4, $\Gamma_{D_{2n}}$ is $(m + 2)$-partite. Thus $(m + 2)$-partite and always contain an element with order p_i and p_j with $i \neq j$. Let $ord(x) = p_i$ and $ord(y) = p_j$, so x and y are adjacent. Consequently, we have cycle $e - x - y - e$ then girth of $\Gamma_{D_{2n}}$ is 3. ∎

4. Conclusion
The obtained result shows that the coprime graph of a dihedral group where n is a composite number specifically $n = p_1^{k_1} p_2^{k_2} p_3^{k_3} \dots p_m^{k_m}$, since it forms a multipartite graph or $(m + 2)$-partite graph, it has a girth of 3, a radius of 1, and a diameter of 2. For the next result, it is interesting to see other characteristics of $\Gamma_{D_{2n}}$, like a clique, chromatic numbers, etc.

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