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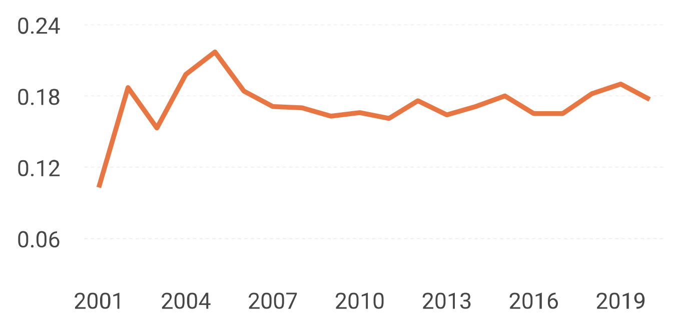

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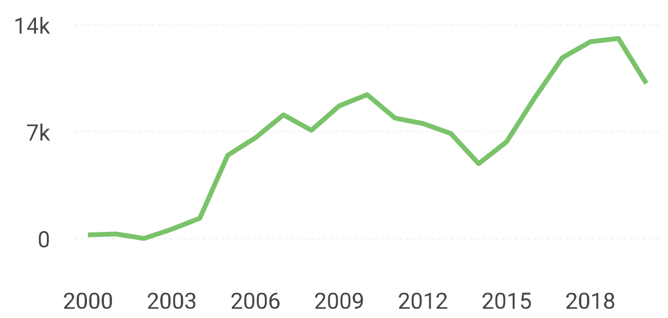

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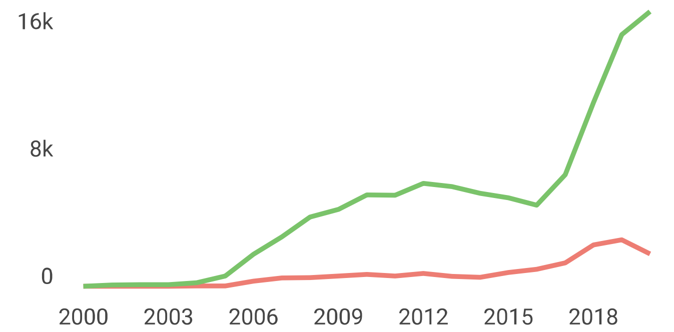

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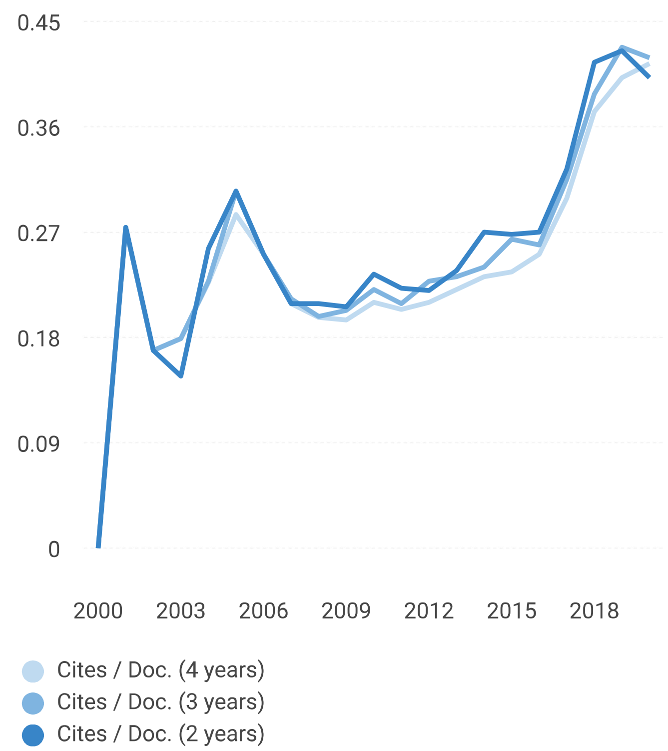

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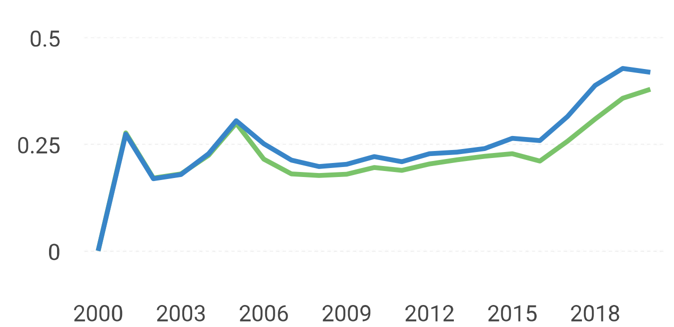

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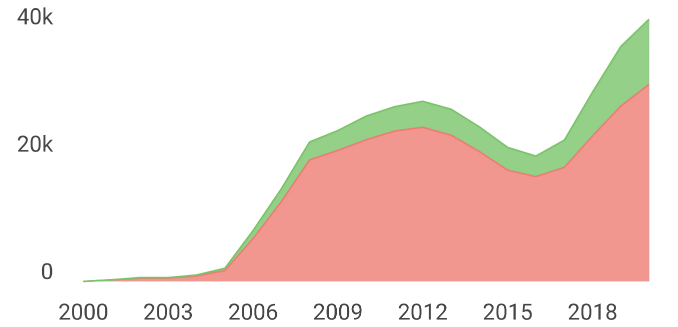

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Some Results of Non-Coprime Graph of the Dihedral Group D_{2n} for n a Prime Power

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Abstract.A graph of a finite group G whose vertices are all elements of G except the identity element, and edges defined as $(u, v) \in E(G)$ if and only if $(|u|, |v|) \neq 1$ is called a non-coprime graph of G and denoted by $\overline{\Gamma_G}$. In this paper we give some properties of non-coprime graphs of a dihedral group D_{2n}, when n is a prime power. One main result of this paper shows that $\overline{\Gamma_G}$ is either a complete graph or can be partitioned into two complete graphs.

INTRODUCTION

The non-coprime graph was first introduced by Mansori [1] who gave some of its characterizations. The non-coprime graph is a dual represetation of the coprime graph that introduced by Ma [2]. Other authors also studied graph representation of groups especially on coprime graph such as on dihedral group by Gazir [3] and on group of integer modulo by Juliana [4].

In this paper we use the results of Gazir [3] on the coprime graph of dihedral group to find its dual representation graph, the non-coprime graph of the dihedral group D_{2n}, where n is a prime power.

Definition 1. [3] A graph is an ordered set(V, E) comprising:

i. The set V is non-empty set of vertices
ii. The set E is edge set of a pair vertices, $E \subseteq \{(v, w) \in V^2\}$

Two vertices $v_{i,} v_j$ are said to be neighbors or adjacent if $(v_{i,} v_j) \in E$. A graph is called an undirected graph if $(x, y) = (y, x)$ for every $(x, y) \in E$, and called a simple graph if every edge $(x, y) \in E$ is unique and $x \neq$. In this paper we only consider an undirected simple graph.

Definition 2. [3] An undirected simple graph $G = (V, E)$ is called a complete graph if for every $x, y \in V$, then $(x, y) \in E$.

A mathematical system with a binary operation is called a group if it satisfies four conditions, namely closure, associativity, identity and invertibility.

Definition 3. [5] A nonempty set G is said to be a group if in G there is defined a binary operation, called the product and denoted by $*$ such that, for any $a, b, c \in G$ then

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i. $a, b \in G$ implies that $a * b \in G$ (closed).
ii. $a * (b * c) = (a * b) * c$ (associative law).
iii. There exists an element $e \in G$ such that $a * e = e * a = a$ for all $a \in G$ (the existence of an identity element).
iv. For every $a \in G$ there exists an element $a^{-1} \in G$ such that $a * a^{-1} = a^{-1} * a = e$ (the existence of inverse).

If $(G.*)$ is a group that satisfies a commutative property, that is for any $a, b \in G$, we have $a * b = b * a$, then $(G.*)$ is called a commutative group or an abelian group. A non-empty subset H of a group G is said to be a subgroup of G if H is a group under the same operation on G. If $|H|$ is finite, it is easy to check that whether H is a subgroup or not. But if $|H|$ is infinite, then we can check whether H is a subgroup or not using the following theorem.

Theorem 1. [5] Let H be a non-empty subset of a group G. Subset H is a subgroup of G if and only if $ab^{-1} \in H$, for any $a, b \in H$.

A way to represent a group into a graph is by describing it by the order of every element of the group. The order of a group's element is given by the following definition.

Definition 4. [5] Suppose $(G.*)$ is any group. Let a be any element of G. The smallest positive integer m that satisfies $a^m = e$ is said as an order of a, and denoted as $|a| = m$.

One of the most interesting groups is dihedral group, which is a group of symmetries of a regular polygon consisting of rotations and reflections. Dihedral groups are playing an important role in group theory, geometry, and chemistry.

Definition 5. [5] The dihedral group with order $2n$, denoted by D_{2n} is the set:
$D_{2n} = \{e, a, a^2, \dots, a^{n-1}, b, ab, a^2b, \dots, a^{n-1}b | a^n = b^2 = e, a^{-1} = bab\}$for n $\geq$ 3.

By definition, we can find the order of every element of the dihedral group, depending on n. For any natural number n, there always exists an element with order 2.

Theorem 2. [5] Let $D_{2n} = \{e, a, a^2, \dots, a^{n-1}, b, ab, a^2b, \dots, a^{n-1}b | a^n = b^2 = e, a^{-1} = bab\}$, then$|b| = |ab| = \cdots = |a^{n-1}b| = 2$

Gazir and Wardhana [5] found the characterizations of subgroup of a dihedral group in the following Theorems:

Theorem 3. [5] Let D_{2n}be the dihedral group with $n \geq 3$. If $S = \{e, a, a^2, \cdots, a^{n-1}\} \subseteq D_{2n}$ then S is a nontrivial subgroup of D_{2n}.

Theorem 4. [5] Let D_{2n}be the dihedral group with $n \geq 3$. If $S = \{e, a^i b\} \subseteq D_{2n}$ where $i = 0,1,2,\cdots, n-1$ then S is a non-trivial subgroup of D_{2n}.

Theorem 5. [5] Let D_{2n}be the dihedral group with $n \geq 3$. If n is composite where$n = p_1 p_2 \cdots p_k$ then $S = \{e, a^{p_i}, a^{2p_i}, \cdots, a^{n-p_i}\} \subseteq D_{2n}$ is a non-trivial subgroup of D_{2n}.

Mansori [1] gave the definition of non coprime graph of a group based on the order of every element of the group. We denote that the order of element x of a group as $|x|$ (look at Def 4).

Definition 6. [3] Let Gbe a finite group. Non-coprime graph of group G, denoted by $\overline{\Gamma_G}$ is a graph with its vertices consist of $G - \{e\}$ and two different vertices u,$v$are said to be adjacent if$(|u|, |v|) \neq 1$.

MAIN RESULTS

The non-coprime graph of D_{2n} is a complete graph or can be partitioned into two complete graphs whenever n is a prime power.

If n is a power of even prime then the non-coprime graph of D_{2n} is a complete graph, as shown in the following theorem.

Theorem 6. Let D_{2n}be the dihedral group. If $n = 2^m$ for some $m \in \mathbb{N}$, then $\overline{\Gamma_{D_{2n}}}$ is a complete graph.

Proof. Since $n = 2^m$, then we have the order of D_{2n} is 2^{m+1}, hence the order of every non-identity element of D_{2n} must be divided by 2.Then for any non-identity $x, \quad \in D_{2n}$ we have $(|x|, |\quad|) = 2^k$ for some $k \in \mathbb{N}$, hence x, y are neighbors, then $\overline{\Gamma_{D_{2n}}}$ is a complete graph. □

If n is a power of an odd prime then the non-coprime graph of D_{2n}can be partitioned into two complete graphs.

Theorem 7. Let D_{2n}be the dihedral group. If $n = p^m$ for some $m \in \mathbb{N}$ and p is an odd prime. Then $\overline{\Gamma_{D_{2n}}}$ can be partitioned into two disjoint complete graphs.

Proof. Let split $D_{2n} - \{e\}$ into two disjoint sets $G_1 = \{a, a^2, \dots, a^{p^m - 1}\}$ and $G_2 = \{b, ab, \dots, a^{p^m - 1}b\}$. We have p divides the order of any $x \in G_1$ and 2 divides the order of any $y \in G_2$ **since x and y are rotation and reflection elements, respectively**. So we have every **two elements** in G_i are neigbors for $i = 1,2$. Since p is odd prime then for any $x \in G_1$ and $y \in G_2$, we have $(|x|, |y|) = 1$. Hence x and cannot be **neighbors**, Therefore $\overline{\Gamma_{D_{2n}}}$ can be partitioned into two disjoint complete graphs.□

Subgroups of dihedral groups can be grouped into two types, that are trivial subgroups and non-trivial subgroups. Obviously, the graph from a trivial subgroup of D_{2n}satisfies the previous theorem.

These are all non-trivial subgroups of dihedral groups according to Gazir and Wardhana [5].

1. $S = \{e, a, a^2, \cdots, a^{n-1}\}$
2. $S = \{e, a^i b\}$ where $i = 0,1,2,\cdots, n-1$
3. $S = \{e, a^{p_i}, a^{2p_i}, \cdots, a^{n-p_i}\}$ where $n = p_1^{k_1} p_2^{k_2} \cdots p_m^{k_m}$
4. $S = \left\{e, a^{\{\prod_{\{i=1\}}^{\{t\}} p_{j_i}\}}, \cdots, a^{\{n - \prod_{\{i=1\}}^{\{t\}} p_{j_i}\}}, a^q b, a^{\{q + \prod_{\{i=1\}}^{\{t\}} p_{j_i}\}} b, \cdots, a^{\{q + n - \prod_{\{i=1\}}^{\{t\}} p_{j_i}\}}\right\}$, where $1 \leq t \leq m$ and $0 \leq q \leq n-1$.

It is easy to check that when n is a prime then all the non-trivial subgroups are only the first two subgroups. In general, the non-coprime graph of any subgroup of $D_{2n}$is either a trivial graph or a complete graph.

Theorem 8. Let S be a non trivial subgroup of dihedral group D_{2n}. If $n = p^m$ then the non-coprime graph of S is a trivial graph or a complete graph or can be partitioned into two complete graphs.

Proof. Obviously for $S = \{e, a^i b\}$where $i = 0,1,2,\cdots, n-1$, $\overline{\Gamma_S}$ is a trivial graph.
So we have three cases left.

Case 1 : $S = \{e, a, a^2, \cdots, a^{n-1}\}$
The order of any non-identity element of S must be divided by p, hence every non identity element of S must be neighbors. Then $\overline{\Gamma_S}$ is a complete graph.

Case 2 : $S = \{e, a^{p^i}, a^{2p^i}, \cdots, a^{n-p^i}\}$ where $i = 1,2,\cdots, m$.
Similar to case 1, we have the order of any non-identity element of S must be divided by p, then we can conclude that $\overline{\Gamma_S}$ is a complete graph.

Case 3 : $S = \left\{e, a^{\left\{\prod_{\{i=1\}}^{\{t\}} p_{j_i}\right\}}, \cdots, a^{\left\{n-\prod_{\{i=1\}}^{\{t\}} p_{j_i}\right\}}, a^q\, b, a^{\left\{q+\prod_{\{i=1\}}^{\{t\}} p_{j_i}\right\}} b, \cdots, a^{\left\{q+n-\prod_{\{i=1\}}^{\{t\}} p_{j_i}\right\}}\right\}, 1 \leq t \leq m$ and $0 \leq q \leq n-1$.

If $p = 2$, obviously the order of every non-identity element of S must be divided by 2, hence we have $\overline{\Gamma_S}$ is a complete graph. If p is an odd prime then the order of non-identity element of elements of S is either 2 or divided by p. Hence we can partition $\overline{\Gamma_S}$ into two complete graphs.□

CONCLUSIONS

Given a dihedral group D_{2n} with n is a prime power, then the non-coprime graph of $D_{2n}$is always a complete graph or can be partitioned into two complete graphs. The same case happened to any subgroups of D_{2n}.

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