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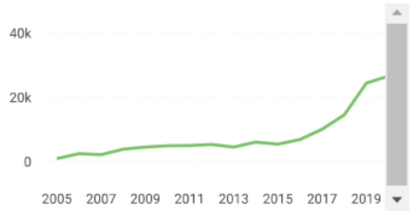

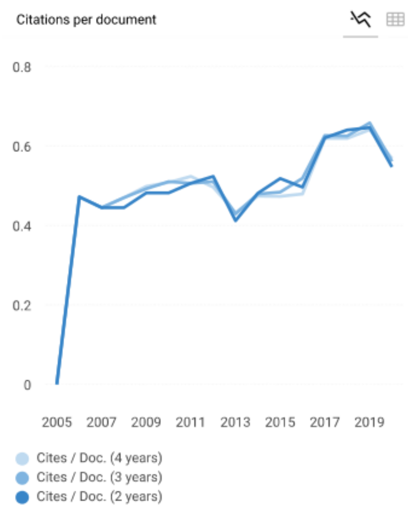

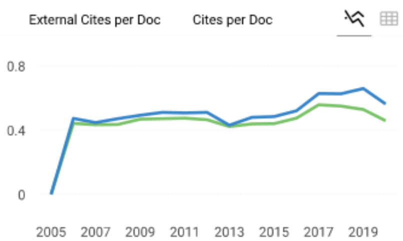

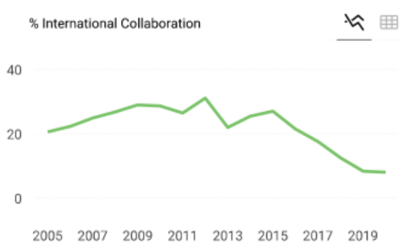

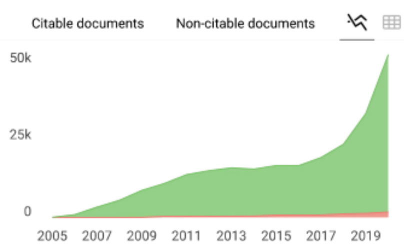

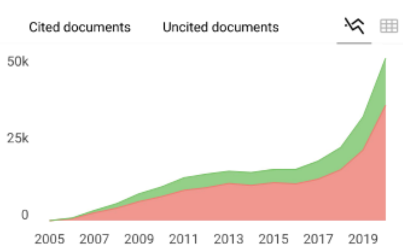

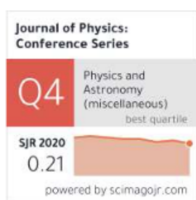

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A note on almost prime submodule of CSM module over principal ideal domain

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A note on almost prime submodule of CSM module over principal ideal domain

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Abstract. An almost prime submodule is a generalization of prime submodule introduced in 2011 by Khashan. This algebraic structure was brought from an algebraic structure in ring theory, prime ideal, and almost prime ideal. This paper aims to construct similar properties of prime ideal and almost prime ideal from ring theory to module theory. The problem that we want to eliminate is the multiplication operation, which is missing in module theory. We use the definition of module annihilator to bridge the gap. This article gives some properties of the prime submodule and almost prime submodule of CMS module over a principal ideal domain. A CSM module is a module that every cyclic submodule. One of the results is that the idempotent submodule is an almost prime submodule.

1. Introduction

The concept of the almost prime submodule is a generalization of a prime submodule. Both concepts are studied in module theory and are seen as a dual concept that is already established in ring theory and known as prime ideal and almost prime ideal. The term prime ideal was introduced by Dedekind in 1897 [1], and later in 2009, Bhatwadekar and Sharma are given the generalization of the prime ideal [2]. Furthermore, after the prime submodule was introduced in 1978 by Dauns [3], Khashan generalized and named it an almost prime submodule in 2012 [4].
Recently, in 2021, a study in both ring theory and module theory gave some properties of almost prime ideal and almost prime submodule. Misuki et al. l. studied cyclic almost prime ideal on Gaussian integer modulo ring [5], and Juliana et al. l studied cyclic almost prime submodule on Gaussian integer modulo module over principal ideal domain [6]. Both of the studies are based on an article in 2019 that stated that prime ideal and almost prime ideal are equivalent in Gaussian integer ring [7].
This article gives some properties of almost prime submodule on CSM module over a principal ideal domain. We used some results from studies conducted by Wardhana et al. l in 2016 on almost prime submodule over principal ideal domain [8][9][10].

2. Almost Prime Submodule of CSM Module over a Principal Ideal Domain

In this paper all ring is commutative with identity and denoted by R, if a ring is a principal ideal domain, we denote it as . The definition of CSM module and multiplication module is given bellow
Definition 2.1. Let M is a module over a ring R. If every submodule N of M is cyclic, then M is called a CSM module.

Definition 2.2. Let M is a module over a ring R. If every submodule N of M, there exists an ideal I with R such that $N = IM$, then M is called a multiplication module.
Since the introduction of ring theory and module theory, the abstraction concept of prime number in both algebraic structures is given bellow

Definition 2.3. Let R be a ring with a proper ideal I. The ideal I is called a prime ideal if for all $x, y \in R$ such that $xy \in I$, then $x \in I$ or $y \in I$.

As we can see, the definition of prime ideal requires a multiplication operation that lacks a module structure. Hence we need to define a fraction of the submodule over the module. If N is a submodule of a module M over a ring R, then $(N: M) = \{r \in R | rM \subseteq N\}$. We have noted that $(N: M)$ is an ideal of R.

Definition 2.4. Let M is a module over a ring R with proper submodule N. The submodule N is called a prime submodule of M if for all $r \in R$ and for all $m \in M$ such that $rm \in N$, then $r \in (N: M)$ or $m \in N$.

It is easy to check that if the module M is R itself, then $(N: M) = N$ and the definition of a prime submodule is equivalent to the definition of a prime ideal.

Now the definition of almost prime ideal and almost prime submodule is given below.

Definition 2.5. Let R be a ring with a proper ideal I. The ideal I is called an almost prime ideal if for all $x, y \in R$ such that $xy \in I - I^2$, then $x \in I$ or $y \in I$.

Definition 2.6. Let M is a module over a ring R with proper submodule N. The submodule N is called an almost prime submodule of M if for all $r \in R$ and for all $m \in M$ such that $rm \in N - (N: M)N$, then $r \in (N: M)$ or $m \in N$.
Like the case of prime ideal and prime submodule, if the module $M = R$, we have an equivalent concept once again.

2.1. A Free CSM Module over a Principal Ideal Domain

A submodule N of a free CSM Module over a principal ideal domain should be a cyclic submodule, and we can write the submodule as $N = k < x >$ for $\{x\}$ is a basis of M.

Let us recall a theorem that given by Wardhana [8].

Theorem 2.1 [8]. Let M be a free module over a principal ideal domain D, and N a proper submodule such that $rank_D(N) = rank_D(M) = n$. Then the following statement is equivalent.
The submodule N is an almost prime submodule of M
There exist a prime number $p \in D$, and a basis of M, $S = \{x_1, \dots, x_n\}$, for which $\{px_1, \dots, px_k, x_{k+1}, \dots, x_m\}$ is a basis of N.

Theorem 2.2 [8]. Let M be a free module over a principal ideal domain D, and N a proper submodule such that $rank_D(N) = rank_D(M) = n$. Then N is almost prime submodule if and only if N is a prime submodule.
Since in our case, the module M is a CSM module, we have n is equal to 1, then we have this theorem.

Theorem 2.3. Let M be a free CSM module over a principal ideal domain D with basis $\{a\}$. A proper submodule N of M is an almost prime submodule if and only if $N =< px >$ for p is a prime number in D and $\{x\}$ is a basis of M.

Proof. Since the rank of the submodule N is equal to the rank of the module M, then based on Theorem 2.1, we proved the theorem.

Theorem 2.4. Let M be a free CSM module over a principal ideal domain D. A proper submodule N of M is an almost prime submodule if and only if N is a prime submodule of M.

We can conclude that the concept prime submodule and almost prime submodule on a free CSM module over a principal ideal domain is equivalent.

2.2. A Primary CSM Module over a Principal Ideal Domain.

A primary CSM module over a principal ideal domain is a module with order a prime power. Since a primary module can be decomposed to its cyclic submodules, if given the module $M =< x >$ with order p^e, then we have its submodule is $N =< p^f x >$ with $f \leq e$.

Theorem 2.5. Let $M =< x >$ be a primary CSM module over a principal ideal domain D with order p^e. If $N =< p^f x >$ is a proper submodule of M then $(N: M) = p^f$.

Proof. Let $r \in < p^f >$, then $r = \beta p^f$ for $\beta \in D$. For all $m \in M$ we have $rm = r\alpha x = \beta p^f \alpha x = \alpha\beta p^f x \in N$. Therefore $< p^f > \subseteq (N: M)$. Conversely, let $r \in (N: M)$, then $rm \in N$ for all $m \in M$. Hence $rx = \gamma p^f x$ for $\gamma \in D$ and clearly $p^f | r$. Therefore $(N: M) =< p^f >$.

Based on Theorem 2.5, we have this result for a prime submodule

Theorem 2.6. Let $M =< x >$ be a primary CSM module over a principal ideal domain D with order p^e. A non-trivial submodule N of M is a prime submodule if and only if $N =< px >$.

Proof. Let $N =< px >$, by Theorem 2.5 we have $(N: M) =< p >$. Let $r \in D$ and $m = \alpha x \in M$ such that $rm \in N$. Then we have $rm = r\alpha x = \beta px$ for $\alpha, \beta \in D$, hence $p | r\alpha$. Since p is a prime number, then $p | \alpha$ or $p | r$. Therefore $m \in N$ or $r \in < p >$, hence N is a prime submodule. Conversely, let N prime submodule. Since N is non-zero, then we have $N =< p^f x >$, and based on Theorem 2.5, we have $(N: M) =< p^f >$. Suppose that $f > 1$ then we have $p \notin (N: M)$ and $p^{f-1} x \notin N$ such that $pp^{f-1} x \in N$ (contradiction). Therefore, since N is proper, we have $f = 1$.

Now we have the equivalent of the prime submodule and almost prime submodule on the following theorem

Theorem 2.7. Let $M =< x >$ be a primary CSM module over a principal ideal domain D with order p^e. A non-trivial submodule N of M is a prime submodule if and only if N is an almost prime submodule.

Proof. We need only to show that if N is an almost prime submodule, then $N =< px >$ for p is a prime number in D. But it is easy to see that if $N =< p^k x >$ for $k > 1$, then we have a contradiction (see the proof in Theorem 2.6). Hence, the concept of the almost prime submodule is equivalent to the concept prime submodule.

2.3. A Multiplication CSM Module over a Principal Ideal Domain

In general, an almost prime submodule need not be a prime submodule. In previous results, we found that an almost prime submodule is equivalent to a prime submodule. Wardhana and Astuti have an example of an almost prime submodule that is not a prime submodule in the multiplication module $\mathbb{Z}$-module $\mathbb{Z}_{12}$ [9]. Submodule $< 4 >$ is an almost prime submodule that is not a prime submodule.

Definition 2.7. Let M a module over a ring R, and N is a submodule of M. Submodule N is called an idempotent submodule if $N = (N: M)N$.

We found that an idempotent submodule is an almost prime submodule.

Theorem 2.9. Let M be a CSM module over a Principal Ideal Domain. If N is an idempotent submodule, then N is an almost prime submodule.

Proof. By definition, an idempotent submodule is submodule N such that $N = (N: M)N$, then we have $N - (N: M)N = \emptyset$. Clearly, N is an almost prime submodule.

The converse of Theorem 2.9 seems to be true since all the examples said so. But we need to work on the proof.

Conclusion

In a CSM module over a principal ideal domain, there is no difference between a prime submodule and almost prime submodule for most cases. This result is analog with the result given by some authors [5][6][7]. But in general, an almost prime submodule needs not to be a prime submodule [9].

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