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Some Characteristics of Prime Cyclic Ideal On Gaussian Integer Ring Modulo

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Abstract. Gaussian modulo integer is a complex number $a + ib$ where $a, b \in \mathbb{Z}_n$. Some of the characteristics of the prime ideal on Gaussian integer are a trivial ideal $\{0\}$ is a prime ideal, and I ideal prime if and only if I almost prime ideal. These characteristics do not necessarily apply to modulo Gaussian integers. In this paper, we give some characteristics of the prime ideal on modulo Gaussian integer.

1. Introduction

Prime numbers always have been an exciting topic among mathematicians until today. The abstraction of prime numbers first introduced by Dedekind in 1871, called prime ideal [1]. Recently, some mathematicians have brought the prime ideal generalization study around the world in different algebraic structures.

Bhatwadekar and Sharma (2009) introduced a generalization of a prime ideal in ring theory, called almost prime ideal [2]. Darani (2011) give more detailed properties around almost prime ideal [3] and new insight about almost prime ideal [4]. Maulana et al. (2019) study prime ideal and almost prime ideal properties on Gaussian integer [5]. Yuwaningsih (2020) give more general properties of almost prime submodule [6]. Additional, Khashan and Bani-Ata (2021) gives a prime ideal version in module theory in other algebraic structures, called almost prime submodule [7].

This study is based on Maulana's research in 2019, aiming to find the prime cyclic prime ideal properties on a particular ring called Gaussian integer modulo.

2. Methodology

The methodology of this research is by making a pattern from some example of the dihedral group for some n, and the conjecture will be made by the pattern. And by deductive, we will prove the conjecture. If the conjecture is correct, then we state it as a Theorem. If wrong, then we make another conjecture.

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3. Results and Discussion

Definition 1. [9] A non-empty set R with two binary operations, addition $(+)$ and multiplication $(\times)$ defined on R is called a ring if it satisfies the following axioms.

i. $(R, +)$ is a commutative group.
ii. The $\times$ operation is associative, that is $(a \times b) \times c = a \times (b \times c)$ for all $a, b, c \in R$.
iii. The distributive law applies to R, that is: for all $a, b, c \in R$ then $a \times (b + c) = (a \times b) + (a \times c)$ and $(a + b) \times c = (a \times c) + (b \times c)$.

Ring R is said to be commutative if the operation $\times$ is commutative, i.e., $a \times b = b \times a$, for every $a, b \in R$. Ring R is said to have unity to the multiplication operation if there is an element $1 \in R$ so that $a \times 1 = 1 \times a = a$, for every $a \in R$.

In a ring with unity, all the elements do not have to have an inverse to multiplication. Elements that have an inverse to multiplication are called units.

The ideal is an essential sub-structure of a ring, the ideal concept of a ring is analogous to the normal subgroup concept of a group.

Definition 2. [9] Let R is a ring, a non-empty subset I of R called ideal if

i. The set I is a subgroup of R, that is $a, b \in I$ applies $a - b \in I$.
ii. The set I closed to the multiplication by each element in ring R, that is, if $a \in I$, $r \in R$ then applies $ar \in I$ and $ra \in I$.

Theorem 1. [9] If R a ring with unity and I is an ideal of R containing a unit, then $I = R$.

The abstraction of prime numbers in the ring was introduced by Dedekind in 1871, called the ideal prime. The following theorem gives it.

Definition 3. [9] An ideal $I \neq R$ in the commutative ring R is a prime ideal if $ab \in I$ implies $a \in I$ or $b \in I$ for $a, b \in R$.

Furthermore, in 2009, the prime ideal was generalized to the ideal almost prime by Bhatwadekar. The definition is as follows.

Definition 4. [2] An ideal $I \neq R$ in the commutative ring R is an almost prime ideal if $ab \in I - I^2$ implies $a \in I$ or $b \in I$ for $a, b \in R$.

Complex numbers are a generalization of real numbers, as well as Gaussian integers, which are the generalization of integers.

Definition 5. [9] Gaussian integer is complex number $a + ib$ where $a, b \in \mathbb{Z}$.

Maulana found the characterizations of Gaussian prime numbers in the following theorem.

Teorema 2. [5] Let $\alpha = a + ib \in \mathbb{Z}[i]$, α satisfies one of the following properties

i. For $a \neq 0,\ b = 0$, a prime number in $\mathbb{Z}$ and $|a| \equiv 3 \ (mod\ 4)$.
ii. For $a = 0,\ b \neq 0$, b prime number in $\mathbb{Z}$ and $|b| \equiv 3 \ (mod\ 4)$.
iii. For $a,\ b \neq 0$, $a^2 + b^2$ is a prime number in $\mathbb{Z}$.

If and only if α is Gaussian prime number.

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Definition 6. [1] Modulo Gaussian integer is a complex number $a + ib$ where $a, b \in \mathbb{Z}_n$.

Every non-zero ring R has at least two ideals, the improper ideal R, and a trivial ideal $\{0\}$. The other ideal in R, we named it proper non-trivial ideal. By definition of prime ideal, it is clear the improper ideal R is not prime ideal, and then a trivial ideal $\{0\}$ is not necessarily prime ideal on modulo Gaussian integer [10]-[11].

A trivial ideal $\{0\}$ is prime ideal when modulo n is Gaussian prime numbers, as shown in the following theorem.

Theorem 3. Let $I = \langle \bar{a} \rangle$ ideal of $\mathbb{Z}_n[i]$. $(a, n) = 1$ if and only if $I = \mathbb{Z}_n[i]$.

Proof. $(\Rightarrow)$ Because $(a, n) = 1$ then there are $p, q \in \mathbb{Z}[i]$ so that $pa + qn = 1$ or $pa = 1 - qn$. By dividing the two sides with n, obtained $\overline{pa} = \bar{1} - \overline{qn}$ or $\overline{pa} = \bar{1} - \bar{0}$. Thus $\bar{1} = \overline{pa}$, so that $\bar{1} = \bar{p}(\bar{a}) \in \langle \bar{a} \rangle$. Because $\bar{1}$ is unit in $\mathbb{Z}_n[i]$ then by theorem 1 obtained $I = \mathbb{Z}_n[i]$.
$(\Leftarrow)$ Let $I = \mathbb{Z}_n[i]$ and suppose $(a, n) = p \neq 1$. Thus $p|a$ and $p|n$ then $\langle \bar{a} \rangle \subset \langle \bar{p} \rangle$. Karena $p|n$, there $q \in \mathbb{Z}[i]$ so that $n = pq$. From that, $\langle \bar{p} \rangle \neq \mathbb{Z}_n[i]$. Therefore $I \neq \mathbb{Z}_n[i]$. ∎

Some ideal on modulo Gaussian integer is equivalent, following the theorem is given.

Theorem 4. Let $I = \langle \bar{a} \rangle$ ideal of $\mathbb{Z}_n[i]$. If $(a, n) = p$ then $\langle \bar{a} \rangle = \langle \bar{p} \rangle$.

Proof. Let $(a, n) = p$ then $p|a$ and $p|n$. For $\bar{x} \in \langle \bar{a} \rangle$ obtained $\bar{x} = \overline{ka}$ or $\bar{x} - \overline{ka} = \bar{0}$ for $k \in \mathbb{Z}_n[i]$. By multiplying the two sides with multiples of n obtained $x - ka = ln$ or $x = ka + ln$ so that $x = krp + lsp = (kr + ls)p$ for $r, s \in \mathbb{Z}_n[i]$. Therefore $\bar{x} = \overline{(kr + ls)}\bar{p} \in \langle \bar{p} \rangle$. Otherwise for $\bar{x} \in \langle \bar{p} \rangle$, then $\bar{x} = \overline{kp}$ or $\bar{x} - \overline{kp} = \bar{0}$. Furthermore, the two sides multiplying with multiples of n so that $x - kp = ln$ or $x = ln + kp$. Because $(a, n) = p$ obtained $ra + sn = p$ for $r, s \in \mathbb{Z}[i]$ obtained $x = ln + k(ra + sn) = ln + kra + ksn$. Therefore $\bar{x} = \overline{kra}$ or $\bar{x} = \overline{kr}(\bar{a}) \in \langle \bar{a} \rangle$. So $\langle \bar{a} \rangle = \langle \bar{p} \rangle$. ∎

Ideal $I = \langle \bar{a} \rangle$ on modulo Gaussian integer is prime ideal when great common divisor n and a is Gaussian prime number, this is given by the following theorem.

Theorem 5. Let $I = \langle \bar{a} \rangle$ non-zero ideal in $\mathbb{Z}_n[i]$. If $(a, n) = p$ is Gaussian prime number then $I = \langle \bar{a} \rangle$ is a prime ideal.

Proof. It is clear $\langle \bar{a} \rangle \neq \mathbb{Z}_n[i]$. Case 1. Suppose a is Gaussian prime number then $a = p$ so that $a|n$ or $n = aq$. Thus for any $\overline{xy} \in \langle \bar{a} \rangle$ then $\overline{xy} = \overline{ka}$. By multiplying the two sides with multiples of n obtained $xy - ka = ln$ or $xy = ka + ln$. Because $n = aq$ then $xy = ka + laq = (k + lq)(a)$. Because a is a Gaussian prime number, it must $a|x$ or $a|y$, obtained $\bar{x} \in \langle \bar{a} \rangle$ or $\bar{y} \in \langle \bar{a} \rangle$. Therefore I is a prime ideal. Case 2. Suppose a is not a Gaussian prime number, and because $(a, n) = p$ then by theorem 4 obtained $\langle \bar{a} \rangle = \langle \bar{p} \rangle$. By case 1, $\langle \bar{p} \rangle$ is a prime ideal. Therefore $\langle \bar{a} \rangle$ also prime. ∎

In the Gaussian ring integer, I is a prime ideal if and only if I is an almost prime ideal. On modulo Gaussian integer, this property does not apply to the opposite, for example, in $\mathbb{Z}_{12}[i]$, $I = \langle \bar{4} \rangle$ is almost prime ideal, but I is not a prime ideal. The following is the theorem given.

Theorem 6. Let $I = \langle \bar{a} \rangle$ non-zero ideal on $\mathbb{Z}_n[i]$. If I is a prime ideal, then I also almost prime ideal.

Proof. Let any $\overline{xy} \in I - I^2$. Because $I - I^2 \subset I$ and I is prime ideal then it is clear $\bar{x} \in I$ or $\bar{y} \in I$. So I is an almost prime ideal. ∎

4. Conclusion

In rings Gaussian Integer Modulo, we have some characterizations of the cyclic prime ideal of $\mathbb{Z}_n[i]$. If given $I = \langle \bar{a} \rangle$, a non-zero ideal in $\mathbb{Z}_n[i]$ and If $(a, n) = p$, where p is a Gaussian prime number, then $I = \langle \bar{a} \rangle$ is a prime ideal. And if I is a prime ideal, then we found that I is almost prime ideal, but it is not the opposite.

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