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3 The Clique Numbers and Chromatic Numbers of The Coprime Graph of a Dihedral Group

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Abstract. The graph has many properties and characterizations. One interesting topic to discuss is the clique numbers and chromatic numbers. This research will determine the clique numbers and chromatic numbers of the coprime graph of the dihedral group. One of the main results is if $n = 2^k$ then the chromatics numbers of the coprime graph of the dihedral group are 2, and if n is odd composite numbers, then the clique numbers and chromatics numbers are $(m + 2)$.

1. Introduction

Many mathematicians study the representation graph from an algebraic structure in recent years. Mostly studied the representation graph from an algebraic structure called group. Another study is the representation graph from an algebraic structure called ring and module [1]. The coprime graph introduced in ref [1], he gives the definitions of a graph as a representation of a group. In ref [2], Dorbini classified is a complete t -partite graph or planar graph and automorphism group of Γ_G . In addition, Mansoori (2016) gives a dual version of the coprime graph called non-coprime [3].

In 2020, two different studies about the coprime graph were given in ref [4]. Syarifudin et al. (2020) studied some characterization of a dihedral group's coprime graph. He gives a sufficient condition whenever the coprime graph forms a complete bipartite graph, a complete tripartite graph, and m -partite graph. He also studies other properties such as radius, diameter, and girth [4], [6]. On the other hand, Juliana et al. (2020) give some characterization of the coprime graph of an integer modulo group [5]. Masriani et al. (2020) give a dual version of Juliana's studied on a non-coprime graph of an integer modulo group [7].

In another study about the representation graph of a group, Chelvama et al. give the clique numbers and chromatics numbers of the dihedral group's commuting graph [6]. Abdollahi et al. previously give some properties of the representation graph of a group called non-commuting graph [9] and non-cyclic graph [10].

This paper aims to find the clique numbers and chromatic numbers of the dihedral group's coprime graph.



2. Methodology

The methodology of this research is by making a pattern from some example of the dihedral group for some n , and the conjecture will be made by the pattern. And by deductive, we will prove the conjecture. If the conjecture is correct, then we state it as a Theorem. If wrong, then we make another conjecture.

3. Result and Discussion

This section will discuss the coprime graph of a dihedral group and some characterization like clique numbers and chromatic numbers. Before that, several definitions that support this paper will be presented.

A dihedral group is a group generated by two elements, more details in the following definition.

Definition 1 ([3]) Group G , named dihedral group with order $2n$, $n \geq 3$ and $n \in \mathbb{N}$, is a group generated by $a, b \in G$ with properties

$$G = \langle a, b | a^n = e, b^2 = e, bab^{-1} = a^{-1} \rangle$$

A dihedral group with order $2n$ denoted by D_{2n} .

The following definition of the order of each element of a group, as follows.

Definition 2 ([3]) If $(G, *)$ any group. Let a any elements of G . The least positive integer m with $a^m = e$ (e identity in G) then m named order of a , and denoted by $|a| = m$.

The following definitions are a point in constructing the coprime graph of a dihedral group.

Definition 3 ([1]) Let G finite group, the coprime graph of G denoted by Γ_G is a graph with vertices are elements of G and two distinct vertices x and y are adjacent if and only if $(|x|, |y|) = 1$.

The following definition describes the clique number of a graph, explained in the next Theorem.

Definition 4 The subset of the vertices of Γ_G is called a clique if the induced subgraph of a subset is a complete graph. The maximum size of a clique in a graph Γ_G is called the clique number of Γ_G and denoted by $\omega(\Gamma_G)$.

This last definition discusses the chromatic number of a graph that is related to graph coloring.

Definition 5 Let $k > 0$ be an integer. A k -vertex coloring of a graph Γ_G is an assignment of k colors to the vertices of Γ_G such that no two adjacent vertices have the same color. The chromatic number $\chi(\Gamma_G)$ of a graph is the minimum k for which Γ_G has a vertex coloring.

2.1 Coprime Graph of D_{2n}

The first Theorem about the form of the coprime graph of D_{2n} , with n odd prime number explained in the next Theorem.

Theorem 1 ([3]) Let n is an odd prime number, then the coprime graph of D_{2n} is complete tripartite.

Proof. Let n is an odd prime number. Define three partition by $V_1 = \{e\}$, $V_2 = \{a, a^2, \dots, a^{p-1}\}$ and $V_3 = \{b, ab, a^2b, \dots, a^{p-1}b\}$. Clearly $|e| = 1$, as n is a prime number. Since the order of a is n and the order of b is 2, then we have $|a| = |a^2| = \dots = |a^{p-1}| = p$, and $|b| = |ab| = |a^2b| = \dots = |a^{p-1}b| =$

2. Hence for each $x, y \in V_i$ then $(|x|, |y|) = p \neq 1$ or $(|x|, |y|) = 2 \neq 1$, $i \in \{2, 3\}$. Then for any $u \in V_i$ and $v \in V_j$ where $i \neq j$ we have $(|u|, |v|) = 1$, thus u and v are adjacent, then the coprime graph of this dihedral group is complete tripartite. ■

The next Theorem explains the coprime graph of D_{2n} is complete bipartite when $n = 2^k$, for some $k \in \mathbb{N}$.

Theorem 2 ([3]) Let $n = 2^k$, for some $k \in \mathbb{N}$ then the coprime graph of D_{2n} is a complete bipartite.

Proof. Let $D_{2n} = \{e, a, a^2, \dots, a^{n-1}, b, ab, \dots, a^{2^k-1}b\}$ is a dihedral group. Define two partitions by $V_1 = \{e\}$, and $V_2 = \{a, a^2, \dots, a^{2^k-1}, b, ab, a^2b, \dots, a^{2^k-1}b\}$. Clearly $|e| = 1$, as Let $n = 2^k$ then we have $|a| = |a^2| = \dots = |a^{2^k-1}| = 2^s$ and $|b| = |ab| = |a^2b| = \dots = |a^{2^k-1}b| = 2$. Hence for each $x, y \in V_2$ then $(|x|, |y|) = 2^s \neq 1$ for some $s \in \mathbb{N}$, thus x and y are not adjacent. Since $|e| = 1$ and for any $v \in V_2$ we have $(|e|, |v|) = 1$. We have e , and v are adjacent, then the coprime graph of this dihedral group is complete bipartite. ■

This Theorem explains about the coprime graph of D_{2n} is complete tripartite for some $n = p^k$, explained in the next Theorem.

Theorem 3 ([3]) Let $n = p^k$ for some $k \in \mathbb{N}$ and p is a prime number, $p \neq 2$, then the coprime graph of D_{2n} is complete tripartite.

Proof. Let $D_{2n} = \{e, a, a^2, \dots, a^{p^k-1}, b, ab, a^2b, \dots, a^{p^k-1}b\}$ is a dihedral group. Define three partitions by $V_1 = \{e\}$, $V_2 = \{a, a^2, a^3, \dots, a^{p^k-1}\}$ and $V_3 = \{b, ab, a^2b, \dots, a^{p^k-1}b\}$. Clearly $|e| = 1$, since $n = p^k$, $p \neq 2$ then we have $|a| = |a^2| = \dots = |a^{p^k-1}| = p^s$ and $|b| = |ab| = |a^2b| = \dots = |a^{p^k-1}b| = 2$. Hence for each $x, y \in V_i$ then $(|x|, |y|) = p^s \neq 1$ for some $s \in \mathbb{N}$ or $(|x|, |y|) = 2 \neq 1$, $i \in \{2, 3\}$. Then for any $u \in V_i$ and $v \in V_j$ where $i \neq j$ then $(|u|, |v|) = 1$. Hence u and v are adjacent, then the coprime graph of the dihedral group is a complete tripartite graph. ■

The last Theorem in this section about the form of the coprime group of D_{2n} where n that the following Theorem gives more generalize.

Theorem 4 ([7]) Let $n = p_1^{k_1} p_2^{k_2} p_3^{k_3} \dots p_m^{k_m}$ where $1 \leq i \leq m$, p_i are distinct prime number, and $p_i \neq 2$ then the coprime graph of D_{2n} is $(m + 2)$ -partite.

Proof. Let D_{2n} a dihedral group with $n = p_1^{k_1} p_2^{k_2} p_3^{k_3} \dots p_m^{k_m}$ where $1 \leq i \leq m$, p_i are distinct prime number, $p_i \neq 2$. We define some set, the first set is a set of elements with order 1, the second set is a set of elements with order 2, or even, the third set is a set of element with order p_1 and odd, and the $(m + 2)^{th}$ set is a set of elements with order p_m and odd and p_j not divide p_m where $1 \leq j \leq m - 1$, clearly these sets are a partition of D_{2n} . Let $x, y \in V_i$, thus $p_i | ord(x)$ and $p_i | ord(y)$, so $(ord(x), ord(y)) \neq 1$, then x and y are not adjacent. So, the coprime graph of D_{2n} is $m + 2$ -partite. ■

2.2 The Clique Number and Chromatic Number

The first Theorem in this section about the clique number of the coprime graph of D_{2n} .

Theorem 5 Let D_{2n} group dihedral. If $\Gamma_{D_{2n}}$ is k -partite graph then $\omega(\Gamma_{D_{2n}}) = k$

Proof. Let $\Gamma_{D_{2n}}$ is k -partite graph, it means that $\Gamma_{D_{2n}}$ has k set of partitions. Let V_1, V_2, \dots, V_k where V_i and V_j adjacent when $i \neq j$ and $1 \leq i, j \leq k$. Because V_i and V_j are adjacent, then we have any $a \in V_i$

and $b \in V_j$ are adjacent so that it is formed complete subgraph C_k in the coprime graph of the dihedral group then $\omega(\Gamma_{D_{2n}}) = k$.

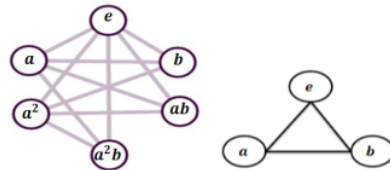


Figure 1. Example of $\omega(\Gamma_{D_{2,3}})$

An example of Theorem 5 can be seen in Figure 1. The next Theorem about the chromatic number of a graph, as follows.

Theorem 6 Let D_{2n} group dihedral. If $\Gamma_{D_{2n}}$ is k -partite graph then $\chi(\Gamma_{D_{2n}}) = k$

Proof. Let $\Gamma_{D_{2n}}$ is k -partite graph, it means that $\Gamma_{D_{2n}}$ has k set of partitions. Let V_1, V_2, \dots, V_k where V_i and V_j adjacent when $i \neq j$ and $1 \leq i, j \leq k$. These V_i and V_j are adjacent, so the color of V_i and V_j must different. Hence for each $a, b \in V_i$, because a and b are not adjacent then a and b have the same color, furthermore each element in V_i have the same color, hence $\chi(\Gamma_{D_{2n}}) = k$.

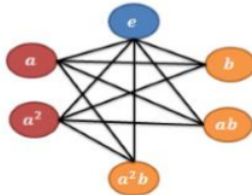


Figure 2 Example of $\chi(\Gamma_{D_{2,3}})$

An example of Theorem 6 can be seen in Figure 2. Some of the corollaries of the preceding Theorem are as follows.

Corollary 1 Let D_{2n} group dihedral. If $n = p_1^{k_1} p_2^{k_2} p_3^{k_3} \dots p_m^{k_m}$ where $1 \leq i \leq m$, p_i are distinct prime number, and $p_i \neq 2$, then $\omega(\Gamma_{D_{2n}}) = \chi(\Gamma_{D_{2n}}) = m + 2$.

Corollary 2 Let D_{2n} group dihedral. If $n = 2^k$, for a $k \in \mathbb{N}$, then $\chi(\Gamma_{D_{2n}}) = 2$.

4. Conclusion

The obtained result shows that the clique number and the chromatic number of the coprime graph of a dihedral group where n composite number specifically $n = p_1^{k_1} p_2^{k_2} p_3^{k_3} \dots p_m^{k_m}$ is $m + 2$ and the clique number & chromatic number of the coprime graph of a dihedral group where $n = 2^k$ is 2.

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