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Understanding Students' Transition from Arithmetic to Algebraic Thinking in the Pre-Algebraic Lesson

Understanding Students' Transition from Arithmetic to Algebraic Thinking in the Pre-Algebraic Lesson

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Abstract. The difficulty in learning algebra is mostly blamed to the discrepancy between arithmetic lesson in elementary school and algebra lesson in middle school. Hence, a bridge is needed to connect those two core subjects thorough a pre-algebraic lesson. This study is aimed to generate an understanding of how the students' shift from arithmetic to algebraic thinking during pre-algebraic lesson. It is a part of a larger study in design research to establish a learning trajectory in pre-algebraic lesson by using pattern investigation. The data were gathered from written works, video registrations and field notes in 32 fifth grade students' of an elementary school in Palembang. The data were analyzed qualitatively using constant comparative method. The result showed that the students' shifting from arithmetic to algebraic thinking is highly impacted by their clarity in seeing geometrical structure of a pattern. Hence, it is recommended for the teachers to conduct a pre-algebraic lessons using pattern investigations visualized in geometrical representation for the elementary school students

1. Introduction

The students in primary school discuss a number of arithmetic topics from types of numbers and the operation within it. Hence, the arithmetical thinking already builds on their cognitive schema. For instance, in arithmetic class, the students learn that 1 is an odd number and 2 is an even number. Also, 3 is odd and 4 is even. Later, the students will be able to notice that the odd and even numbers are occurred one after another.

In higher grade, it will not be sufficient for the students to only have an arithmetical thinking as they also need to see a generalization. For instance in the previous example, if the students want to check whether 97 is odd or even, it will be insufficient for them to list it from 1, 2, 3, etc. until they got 97. Hence, the students need to think further about the relation between the numbers, in a pre-algebraic class, to identify why some numbers are characterized as odd and others are even. The skill is called algebraic thinking abilities.

The crucial aim of learning algebra is to develop the students' algebraic thinking which is defined as the ability to focus on relations between the numbers [1]. However, in Indonesia, this aim is mostly forgotten due to the focus on the learning algebra which tends to merely concern with the symbols, notations and the procedures to find a solution of an equation. The procedural approach on teaching

algebra leads to a number of difficulties to make sense on what is really learn when a student learn about algebra. Hence, it is need to reform the teaching and learning algebra in Indonesia [2].

Historically, algebra is emerged as a generalization of arithmetic. However the students cannot see the relation between them and tend to believe that those two are unrelated knowledge [3]. Therefore, it is necessary to have the students' see the connection between arithmetic and algebra. In line with that it is recommended to conduct a pre-algebraic lesson in preliminary classroom [4] & [5].

The recommended activity in early grades is to use pattern as the pre-algebraic topic [6]. The pattern investigation could be designed in a different form, for instance pictorial or geometrical representations, number patterns, patterns in computational procedures, linear and quadratic patterns and repeating patterns [7].

Early Algebra is essential due to enhance students' flexibility to work with structures [5]. Nonetheless, it is not aimed to transfer the high school curriculum to the elementary one [8]. Some previous studies found that pattern investigation can be helpful for the algebraic activity and therefore recommend this to start algebraic lessons in earlier grades [9], [10], [11] & [12]. Even though a number of studies pointed out the benefits of pattern investigation activities to connect arithmetic and algebra, some other researches argued that the generalization from geometric pattern to algebraic symbolization is too hard for many students [13] & [14]. Hence, the study would like to find the answer of "*how are the students shift from arithmetical to algebraic thinking in a pre-algebraic lesson?*"

2. Methods

This article is a part of a larger study in designing Local Instructional Theory in Pre-Algebraic lesson. It was conducted by using three steps of Design Research, which is Preliminary Study, Teaching Experiment and Retrospective Analysis. Teaching experiment was conducted in 32 students of fifth grade students in an elementary school in Palembang.

The designed learning activities are based on the principles of Realistic Mathematics Education (RME) approach. Based on RME point of view, which is influenced by Freudenthal [15], mathematics should be valued as a human activity. Therefore, instead of transfer the knowledge of mathematics to the learners, RME demand for a rich learning environment which gives the students chance to do mathematics.

The proposed activities used the context of arithmetic sequence in dance formation that is visualized geometrically. The following discussion will show the important of geometric visualization to help the shift from arithmetic to algebraic thinking.

3. Results and Discussion

In the complete study, we employed fives types of arithmetical sequence embodied in geometrical representation. This study will highlight the third activity which is square number pattern. The example of square numbers are 1, 4, 9, 16, 25 and so on. By observing the value, one can notice that the difference between the terms in a square number is an increasing odd number. For instance, from1 to 4 we need to add 3, from 4 to 9 we need to add 5. It can be predict that after 25, you will have 36 since $25 + 11 = 36$. The method of addition in determine the next term of pattern is correct. The ability to see the pattern recursively is one example of arithmetic thinking.

However, as the pattern goes forever, it will be not efficient to calculate it one by one, if we want to look at a "far" term. For example 95th term or 127th term. Hence, we need to think another perspective to see the pattern. If we look at the some examples of the terms, we can see that it follows another rule: it is a square of a number. For instance, 1 is the square 2, 4 is the square of 2, 9 is the square of 3, etc. Hence, it can be predicted that a square number will have a form of n^2. The method of thinking in

a bigger picture is called algebraic thinking. Mathematically speaking, the prediction made during algebraic thinking should be rigorously proven. However, as the lesson is addressed for elementary students, it will be enough until the students can see the structure in general using geometric representation.

Hence, the lesson is guided the students to realized that a square number pattern got its name since it can be visualized geometrically in square shape as in the following Figure 1. In this study, we introduce the idea of square number as the formation created by a flash mob Pencak Silat (Indonesia traditional martial art) performers in *n* formation.

Figure 1. Square Number Pattern

The problem was to draw the 4th formation to continue the sequence given in the Figure 1. There are 3 types of strategy employed by the students, which are addition, grouping and multiplication. The initial strategy is by using addition. In the following Figure 2, the example of the strategy can be observed.
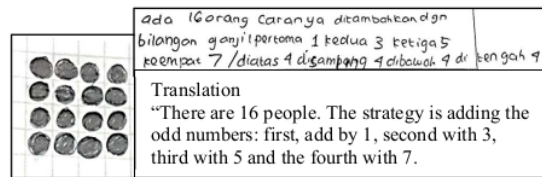

Figure 2. The Addition Strategy

The students' answer in Figure 2 were talking about the additional number added to get the next formation. For instance, the first formation consists of 1 person and they add 3 to get the second formation. Then, from the second formation they add 5 to get the third formation. Hence, they will need to add 7 people to get the 4th formation.

The second strategy was using the grouping addition in which the students see the group of people in the formation. The following Figure 3 is the example of students' work.
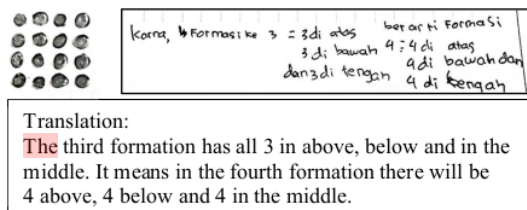

Figure 3. The Grouping Addition Strategy

The grouping addition strategy is a step ahead compared to addition strategy because in this strategy the students started to notice the basic number on each formation. As in the Figure 3, the students realized that there are the rules of 3 in the 3rd formation and rules of 4 in the 4th formation. They were

no longer depends on the total number in the previous formation to predict the number in the nth formation.

The third strategy was multiplication. In this strategy, the students see the relation between the number of people in the rows and the number of people in columns with the total number of people in the certain formation. The example of students' explanation can be seen in the following Figure 4.
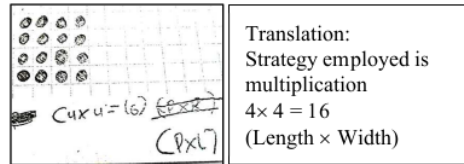

Figure 4. The Multiplication Strategy

The next three questions are to determine the number of dancers in the 10th, 15th and 100th formations. To solve this problem the students used quite similar strategy, which can be classified as addition and multiplication. Even though the strategy given in the Figure 4 is the most sophisticated answer to approach the problem, we figured out that it is not easy for the students to come to that point. In the Fragment 1 and 2 we will show the students' struggle to move from addition to the multiplication strategy. The discussion was happened between Researcher (R) and a group consist of 4 students, S1, S2, S3 and S4.

Fragment 1: Addition Strategy
[1] R   : What strategy you use?
[2] S1 : Addition.
[3] R   : Addition. What would you do if I ask to find the 100th formation? Will you still add the numbers?
[4] S1 : I do.
[5] R   : Really? Can you find another strategy?
[6] S1 : No...
[7] R   : Let's observe the shape of the formation.

Fragment 2: Multiplication Strategy
[1] S1 : Oh this is multiplication! 10 times 10!
[2] S2 : Yes!
[3] S3 : Squaring...
[4] S1 : What squaring are you talking about!
[5]   (*Said something in local language*)
[6] R   : Why? Why?
[7] R   : 10 times 10 is 100, why?
[8] S4 : We find another strategy!
[9] S2 : Since there are 10 rows
[10] S4 : Suppose a row is fulfilled by 10 people and then multiply it with the number of people in that row.
[11] R : There is a row with 10 people and then perform multiplication. Can you explain it more?
[12] S4 : There are 10 rows, in each row there are 10 people. So, 10 times 10
[13] R : So, 10 times 10. How it will be in the 15th formation?
[14] S4 : 15 times 15

From the Fragment 1 we can observe how the students easily give up to find a more efficient way to solve a problem. Here the role of teacher is very important to bridge the gap between students' strategies. Instead of directly express her/his agreement toward particular strategy, the teacher should give the students chance to explore more and to construct more sophisticated idea. It can be done by ask questions which challenge the students to think more and emphasize some important ideas.

In the end of the lesson, the students were able to conclude the square relation between the number of formation and the number of people in that formation. In higher class, the language they used represent the mathematical formula of $N = n^2$, where N is the total number of performers in n^{th} formation.

The students' generalization of the pattern using their own language is an answers of one of the common algebra teaching and learning problems in Indonesia. A number of study found that the students are failed to generate an algebraic formula of certain algebra word problems [16], [17] & [18]. We predicted that it is because the given problem does not provide enough clue to help the students to start working with the structure of the pattern.

As in the pilot study, we found that when the problems were directly asked to "create a general formula", the students lost. But they did generalization when the problems ask them to work with the numbers. Hence, we increased the number to push the students find an efficient way to explain how they solve the problem. Some students also used a symbol to represent their thinking.

Giving the students a chance to work with a pre-algebraic lesson with the support of geometrical pattern is a suitable approach to develop students' thinking ability even though they use a daily language. Later in higher school level, the students' will be able to use a correct mathematical symbol and notation.

In general, as is found in this study, the geometrical representation of arithmetical problem that supports the students in shifting from arithmetic to algebraic thinking can be seen both as context and as a mathematical model. As the context, the geometrical pattern gives the student guidance to start working with the problem. The similar recommendation also given by [19] that students' generalization is influenced by the representation they construct in their cognitive scheme. Hence, the initial task should be made in representation such as figure, instead of the use of symbol [20].

As a mathematical model, the visualization help them to recognize the structure of the series, a skill that tend to be valued as an abstract concept for them if it is directly started with numbers. As the series grows, the students will lay on the important aspect of the structure. Hence, we conclude that in the students' transition from arithmetic to algebraic thinking, is depend on how they see the structure of the pattern. This is called as structure sense [7] which have a relation with personal reference while doing generalization. In line with that, we suggest that the implementation of pre-algebraic activity which is aimed to bridge the transition from arithmetic to algebraic thinking is using the pattern activities which is embodied in the visual representation.

4. Conclusion

Students' shift from arithmetic to algebraic thinking is affected by the ability of the students in seeing the structure of the geometric pattern. Therefore, to support the students in developing algebraic thinking, it is important to choose a helpful context and appropriate visualization.

The geometrical representation of the number pattern plays two important role in the present study: (1) as a visualization of the context and (2) as a mathematical models. Based on the reflection of the

findings, we understand that the students progressively move from arithmetic to algebraic thinking when they able to see the structure of the geometrical pattern. It is, when the students find the "shortcut" to minimize the effort to find the next series of the given pattern.

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