14. Ketut Sarjana

by Ketut Sarjana

Submission date: 24-May-2023 04:09AM (UTC-0500)
Submission ID: 2100718302
File name: C14. Drs. Ketut Sarjana, MS.pdf (1.46M)
Word count: 7115
Character count: 33952


Prisma Sains: Jurnal Pengkajian Ilmu dan Pembelajaran Matematika dan IPA IKIP Mataram
http://ojs.ikipmataram.ac.id/index.php/prismasains/index
e-mail: prismasains.pkpsm@gmail.com
December 2020. Vol. 8, No. 2
p-ISSN: 2338-4530
e-ISSN: 2540-7899
pp. 150-163

Exploration of Student Thinking Process in Proving Mathematical Statements

***Deni Hamdani, Ketut Sarjana, Ratna Yulis Tyaningsih, Ulfa Lu'luilmaknun, J. Junaidi**

Mathematics Education Department, Faculty of Teacher Training and Education, Universitas Mataram. Jl. Majapahit No. 62, Mataram, West Nusa Tenggara 83115, Indonesia.

*Corresponding Author e-mail: deni.math@unram.ac.id

Received: November 2020; Revised: November 2020; Published: December 2020

Abstract

A mathematical statement is not a theorem until it has been carefully derived from previously proven axioms, definitions and theorems. The proof of a theorem is a logical argument that is given deductively and is often interpreted as a justification for statements as well as a fundamental part of the mathematical thinking process. Studying the proof can help decide if and why our answers are logical, develop the habit of arguing, and make investigating an integral part of any problem solving. However, not a few students have difficulty learning it. So it is necessary to explore the student's thought process in proving a statement through questions, answer sheets, and interviews. The ability to prove is explored through 4 (four) proof schemes, namely Scheme of Complete Proof, Scheme of Incomplete Proof, Scheme of unrelated proof, and Scheme of Proof is immature. The results obtained indicate that the ability to prove is influenced by understanding and the ability to see that new theorems are built on previous definitions, properties and theorems; and how to present proof and how students engage with proof. Suggestions in this research are to change the way proof is presented, and to change the way students are involved in proof; improve understanding through routine proving new mathematical statements; and developing course designs that can turn proving activities into routine activities.

Keywords: exploration; proof; theorem; scheme

How to Cite: Hamdani, D., Sarjana, K., Tyaningsih, R., Y., Lu'luilmaknun, U., & Junaidi, J. (2020). Exploration of Student Thinking Process in Proving Mathematical Statements. *Prisma Sains : Jurnal Pengkajian Ilmu dan Pembelajaran Matematika dan IPA IKIP Mataram, 8*(2), 150-163. doi:https://doi.org/10.33394/j-ps.v8i2.3081

https://doi.org/10.33394/j-ps.v8i2.3081

INTRODUCTION

A mathematical statement is not a theorem until it has been carefully derived from previously proven axioms, definitions, and theorems (Bartle & Sherbert, 2011). The theorem is a statement that has been proven based on a predetermined and accepted statement or theorem or axiom and is a logical consequence of axioms that must be proven deductively (Wikipedia contributors, 2020). Theorems represent the subject, the main summary of the material, and are usually formulated after an proving-strategy has been developed, and after innovative ideas have been elaborated in the process of 'throwing ideas around' (Rav, 1999). The proof and theorem are closely related. Proof from a theorem is a logical argument given in accordance with the rules of the deductive system and is often interpreted as justifying the truth of a theorem statement (Rav, 1999), and is a fundamental part of the mathematical thinking process (Devlin, 2003; Hamdani et al., 2020).

In National Council of Teachers of Mathematics (2000) dan Van de Walle et al. (2012), proof combined with reasoning becomes one of the five school mathematics process standards which are teaching programs from Pre-Kindergarten to Grade 12. Hamdani et al.(2020) show that the activity of proving a mathematical statement has begun to be learned from the elementary to the top level, so it is hoped that it can help children decide whether and why the answer is logical, develop the habit of giving arguments, and make investigating activities an integral part of every solving and is a process that can improve understanding of concepts. This is in line with the aim of proof put forward by Juandi (2008) and Hernadi (2013) is to 1) compile facts with certainty, 2) gain understanding, 3) communicate ideas to others, 4) challenge, 5) make something be beautiful, and 6) construct a mathematical theory. Furthermore, Weber (2003) states 7 objectives of proof, namely as 1) explanation, 2) systemization, 3) communication, 4) discovery of new results, 5) justification of a definition, 6) developing intuition, and 7) providing autonomy.

Proof in mathematics must be based on clear statements and definitions, and valid conclusion drawing procedures (Shadiq, 2015). The ability to prove consists of the ability to construct proof and the ability to validate proof (Selden & Selden, 2003; Anwar et al., 2018). Constructing proof includes the ability to use methods of proof, definition axioms, lemmas, and theorems to show the truth of a statement in mathematics. Meanwhile, validating proof includes the ability to criticize proof related to the types of proof that often appear in mathematics (Selden & Selden, 2003). Meanwhile, according to Anwar et al. (2018), construct proof related to the ability to conceptualize images, find local-localized conceptualizations (properties/conclusions related to one part of the image) and global conceptualizations; and validating usually emphasizes the process of linking the relational relationship between local conceptualization and global conceptualization into a series of statements that support propositions/conclusions that will be proven into a series of logical statements.

From these two opinions above Selden & Selden (2003), and Anwar et al. (2018) it can be concluded that constructing proof is the ability to use some previous axioms, definitions, theorems to show the truth of a new mathematical theorem or statement, and validating proof is the ability to relate the relationship between previous axioms, definitions, and theorems logically to confirm the truth of the theorem or new mathematical statements, and obtained by verbally testing steps. The need to understand and especially write proof in mathematics courses is very important, considering that many students say that "I can understand the material, but sometimes I can't do the proof" (Morash, 1987). Agreeing with Morash (1987) and Miyazaki et al.(2017) said that proof is central to mathematics, difficulties in learning and teaching proof are well recognized internationally. From this point of view, it is not surprising to find that there are students who have difficulty writing proof.

Regarding proving a theorem, one of the courses that requires critical thinking used in the deductive process is the real analysis course. Real analysis is a subject that has a big role for mathematics students who want to change difficult routine formulas, because it can develop deductive thinking skills, analyze mathematical situations and expand ideas into new contexts (Bartle & Sherbert, 2011), and is one of the subjects lectures in mathematics are quite strict in enforcing the deductive-axiomatic system (Harini et al., 2014). The use of axioms in proof is an unavoidable choice and proof becomes part of the standard material in real analysis (Botts & Royden, 1964).

In studying real analysis, many students experience difficulties in proving the theorems or mathematical statements contained therein. These difficulties are directly proportional to the number of students who every year or semester return to program and follow the same courses. This phenomenon becomes the basis for exploring the student's thought process in proving and validating a new mathematical theorem or problem, as well as other conceptualizations that may hinder or make it difficult for students to construct and validate evidence.

METHOD

This research is a descriptive qualitative research that aims to explore the thinking process of students in proving (constructing and validating proof) theorems using the assimilation and accommodation framework according to (Subanji, 2006; Netti et al., 2016; Netti & Herawati, 2019). Subjects were taken by giving proof questions to 35 students. The problem of proof in question is

$$\textit{Let } A, B, \textit{ and } C \textit{ are sets. Prove that } A - (B \cup C) = (A - B) \cap (A - C)$$

By adopting a problem structure from (Subanji, 2006; Netti et al., 2017). Each answer sheet for a given proof of evidence is used as the basis for compiling a proof scheme. The proof scheme in Figure 1 below will be used as a guide or comparison for each proof answer sheet that is carried out.

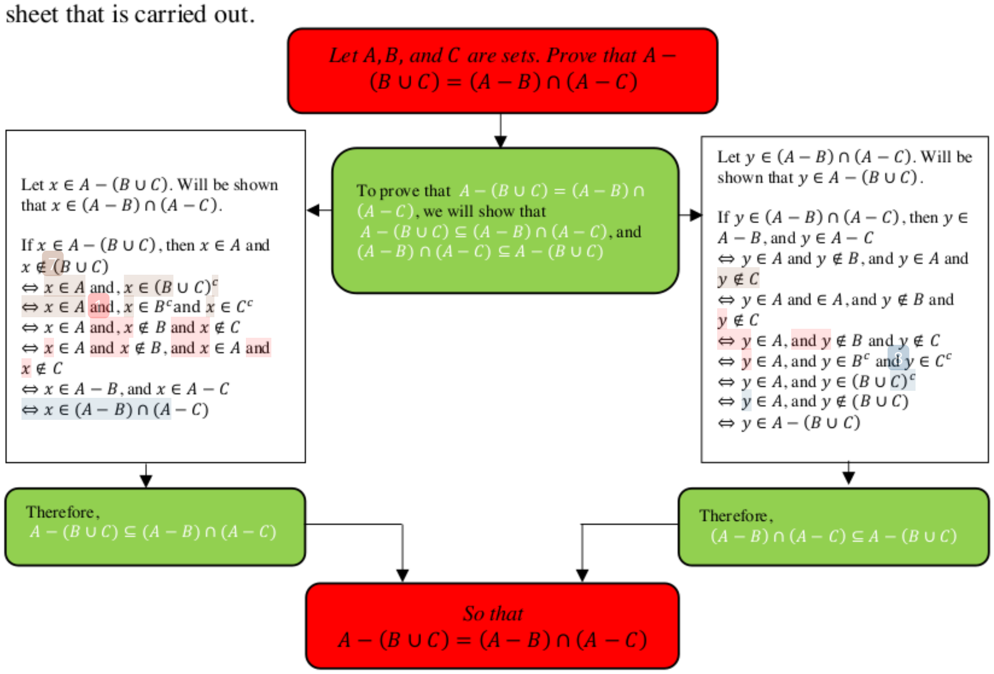

Figure 1. Scheme of Complete proof

The proof provided is not sufficient to ensure that every statement in the argument is true. However, one has to check whether there is any compelling reason to believe that each statement follows from the previous statement (Alcock & Weber, 2005). So that in addition to the answer data for verification questions, data is also collected through interviews and documentation. The interview is based on the response or explanation given by the subject. The recorded interview data will be transcribed to support the interpretation or matching of the written data on the proof answer sheet given. Proof answer sheet data and recorded data are considered valid if they show consistency between written data and interviews. Otherwise, there will be reflection (accommodation) on the lack of and inappropriate schemes.

RESULTS AND DISCUSSION

From the proof questions given to the research subject (students), it shows that 12 of the 35 student proof answers are declared valid and the remaining 23 people are declared invalid. This data indicates that the student's ability to construct proof is still low. The proof answer sheet which is categorized as valid and invalid, will then be described in the following Figure 2, with the aim of providing a concrete picture of the student's ability to prove.

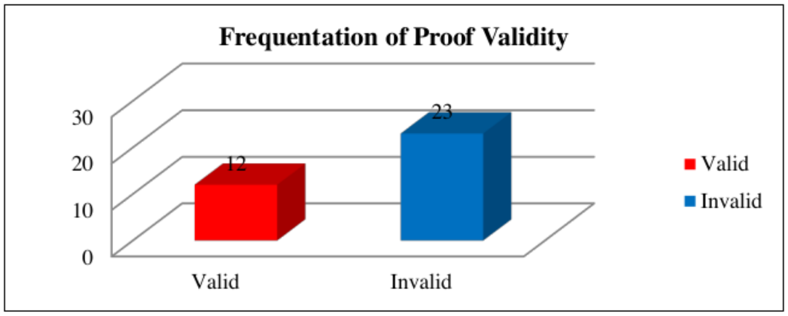

Figure 2. Data of Proof Validity

The concrete images from Fugure 2 above are then re-analyzed and grouped according to the proof scheme. This proving scheme is an adoption of the thinking scheme developed by (Netti et al., 2017). Unlike Netti's thinking scheme, the proof scheme in this paper is divided into 4 (four) schemes, namely 1) complete proof scheme, 2) incomplete proof scheme, 3) unrelated proof scheme, 4) incomplete proof scheme. The four schemes are represented in the following Figure 3.

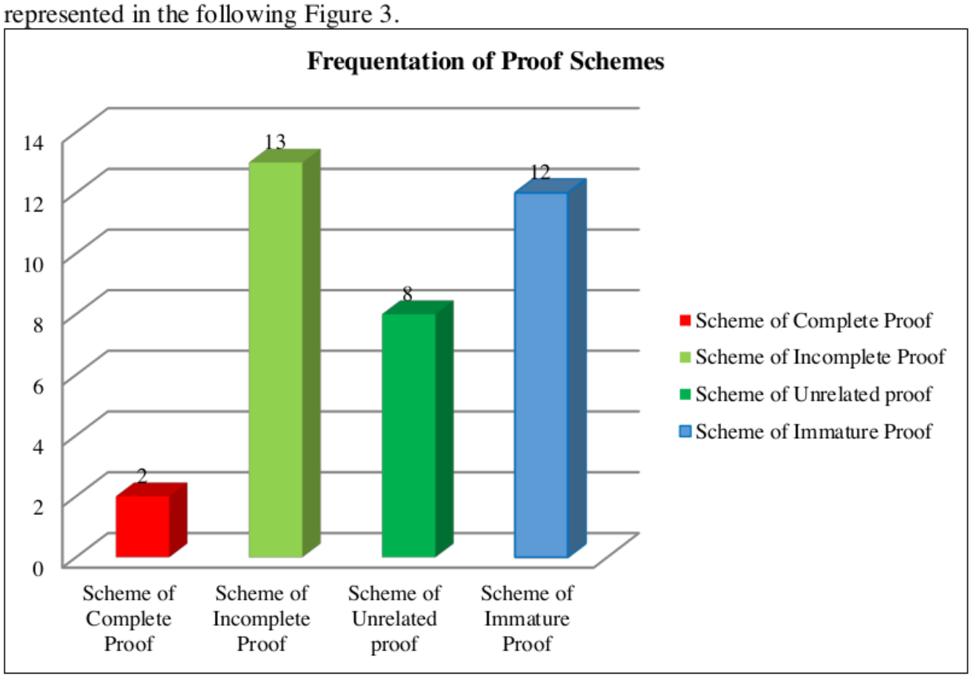

Figure 3. Data of Proof Schemes

The proof scheme data in Figure 3 above shows the least frequency of complete proof schemes (according to the complete proof scheme in Figure 1), and the remaining 13 proof schemes are incomplete, 8 proof schemes are not related, and 12 proof schemes are immature. However, the following will only explain 3 schemes, namely the proof scheme is incomplete, unrelated, and immature. This is because the complete proof scheme is in accordance with the proof scheme.

Scheme of Incomplete Proof

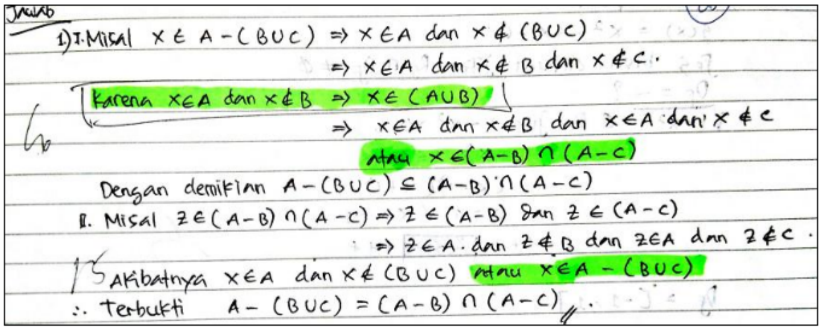

Figure 4. Answer Sheet for AIM subjects

Subjects with incomplete proof consisted of 10 valid and invalid proving subjects as many as 3 subjects, and had construct arguments that were not written based on the diagram proof scheme 1. One of them was AIM (initials). The construction of proof from the subject of AIM is as follows.

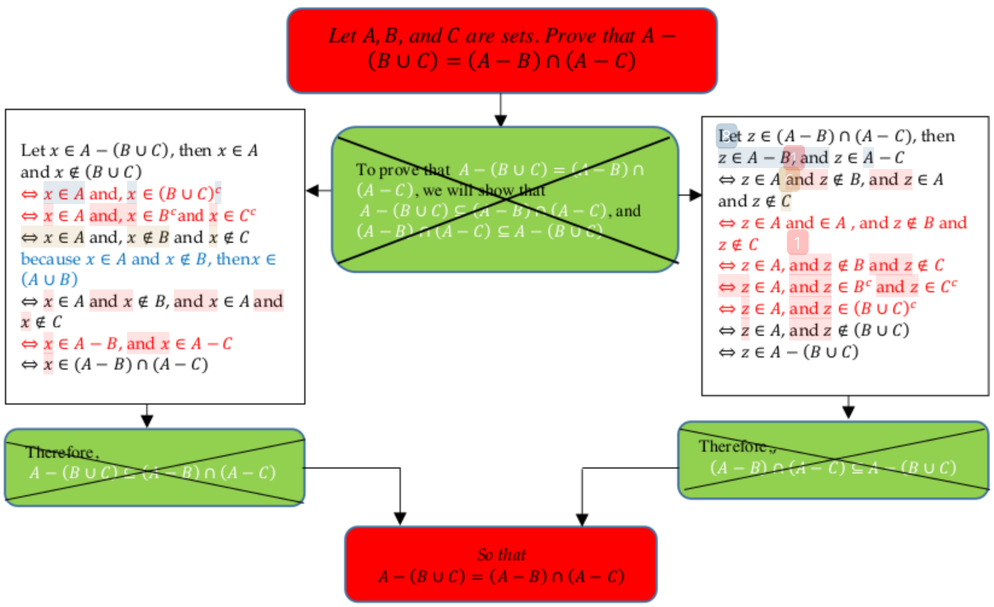

Figure 5. Scheme of Incomplete Proof from AIM subject

The answer sheet for the AIM subject (Figure 4) above is then depicted in schematic form. An incomplete proving scheme of the AIM subject is presented with the following schema. In constructing the proof that $A-(B\cup C)=(A-B)\cap(A-C)$, it seems that the AIM subject does not appear to have written some important ideas/ideas to support the arguments he wrote, so the proof scheme is categorized as incomplete. There are 3 unwriten construct schemes, in constructing the proof that $A-(B\cup C)\subseteq(A-B)\cap(A-C)$, namely 1) $x\in A$ and $x\in(B\cup C)^c$, 2) $x\in A$ and $x\in B^c\cap C^c$, and 3) $x\in A$ and $(x\in B^c$ and $x\in C^c)$, as well as 1 construct error, namely because $x\in A$ and $x\notin B$, then $x\in(A\cup B)$. Furthermore, in the section constructing proof $(A-B)\cap(A-C)\subseteq A-(B\cup C)$ there are 4 unwriten construct schemes, namely 1) $(z\in A$ and $z\in A)$ and $(z\notin B$ and $z\notin C)$, 2) $z\in A$ and $(z\in B^c$ and $z\in C^c)$, 3) $z\in A$ and $z\in B^c\cap C^c$, and 4) $z\in A$ and $z\in(B\cup C)^c$. So it is necessary to reflect (accommodation) through interviews to confirm incomplete or unwritten construct schemes, with the results of the reflections being transcribed in the following conversation.

Table 1. Transcript of interview AIM Subject

Researcher/ Subject		Stimulus or response
Researcher	:	Based on this proving problem: Let A, B, and C are sets. Prove that $A-(B\cup C)=(A-B)\cap(A-C)$. Explain what your understand?
AIM	:	Based on definition $A=B\Leftrightarrow A\subseteq B$ dan $B\subseteq A$, we will show that $\Rightarrow A-(B\cup C)\subseteq(A-B)\cap(A-C)$ $\Leftarrow (A-B)\cap(A-C)\subseteq A-(B\cup C)$
Researcher	:	How do you show that $-(B\cup C)\subseteq(A-B)\cap(A-C)$?
AIM	:	Well, sir. Suppose $x\in A-(B\cup C)$. We will show that $x\in(A-B)\cap(A-C)$
Researcher	:	Okay, now explain your argument to a conclusion $A-(B\cup C)\subseteq(A-B)\cap(A-C)$
AIM	:	Yes, sir. If $x\in A-(B\cup C)\Rightarrow x\in A$ and $x\notin(B\cup C)$ $\Leftrightarrow x\in A$ and $x\notin B$ and $x\notin C$ Because $x\in A$ and $x\notin B$, then $x\in(A\cup B)$ $\Leftrightarrow x\in A$ and $x\notin B$and $x\in A$ and $x\notin C$ Or $x\in(A-B)\cap(A-C)$ So that $A-(B\cup C)\subseteq(A-B)\cap(A-C)$
Researcher	:	Okay, now you write down the complement definition of a set.
AIM	:	Suppose S is the set of universes and $A\subseteq S$. Then $A^c=\{x\mid x\in S$ and $x\notin A\}$
Researcher	:	So, if $x\notin(B\cup C)$, then ...
AIM	:	$x\in(B\cup C)^c$, oh yeah-yeah. So my answer is wrong, sir.
Researcher	:	Now, please add ideas that have not been written, pay attention and understand the previous properties, definitions and theorems.
AIM	:	Yes, sir. So, here it is $x\in A-(B\cup C)\Rightarrow x\in A$ and $x\notin(B\cup C)$ $\Leftrightarrow x\in A$ and $x\in(B\cup C)^c$ $\Leftrightarrow x\in A$ and $x\in B^c\cap C^c$ (De morgan) $\Leftrightarrow x\in A$ and ($x\in B^c$ and $x\in C^c$) $\Leftrightarrow x\in A$ and ($x\notin B$ and $x\notin C$) $\Leftrightarrow$ ($x\in A$ and $x\notin B$) and ($x\in A$ and $x\notin C$) $\Leftrightarrow$ ($x\in A-B$) and ($x\in A-C$) $\Leftrightarrow x\in(A-B)\cap(A-C)$ So that, $A-(B\cup C)\subseteq(A-B)\cap(A-C)$
Researcher	:	Good. Your proof is correct. Now try to improve the proof construction for the argument $(A-B)\cap(A-C)\subseteq A-(B\cup C)$
AIM	:	Yes, sir.

After receiving reflection (accommodation) through interviews, finally the AIM subject was able to show logical proof (according to the proof of diagram Figure 1). It's just that AIM subjects have not realized they have used the idempotent trait when they will show that $(A-B)\cap(A-C)\subseteq A-(B\cup C)$. Furthermore, based on the results of the analysis of the proof answer sheet and the results of interviews and documentation, it can be revealed that theoretically, the incompleteness of the student proof scheme in constructing this proof is at least due to the ability of students to link new information or new ideas with previous ideas, in terms of ideas. The previous ideas cannot be used as material for building new ideas (understanding). The material or previous ideas here can be in the form of axioms, definitions, theorems or properties that can help in constructing a proof.

Scheme of Unrelated Proof

Subjects with unrelated proof schemes are subjects with invalid theorem construction results, which consist of 8 subjects (Figure 3). One of them is BSH (initials). The conclusion of the proof construction of BSH is correct, but the construct schemes used are not interrelated. The proof construction of BSH is as follows.

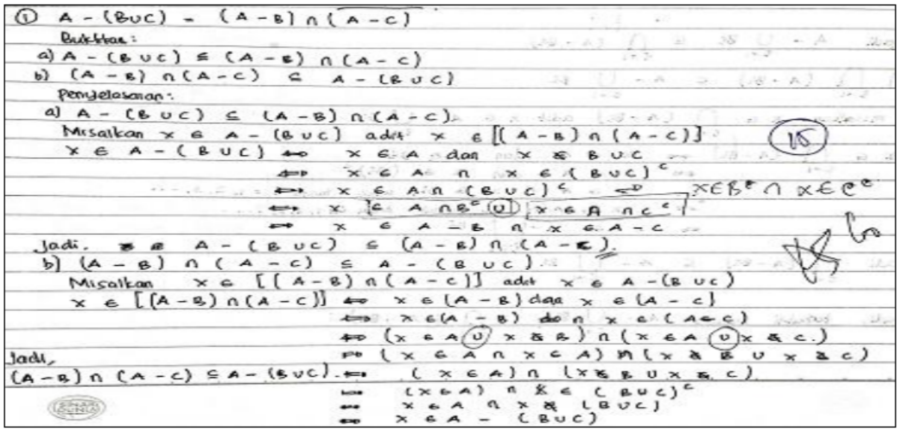

Figure 6. Answer Sheet for BSH subject

The answer sheet in Figure 6 above is the answer sheet from the subject with an unrelated proof scheme, and the answer sheet above if it is described in schematic form, then the unrelated proof scheme from BSH can be presented with the following scheme.

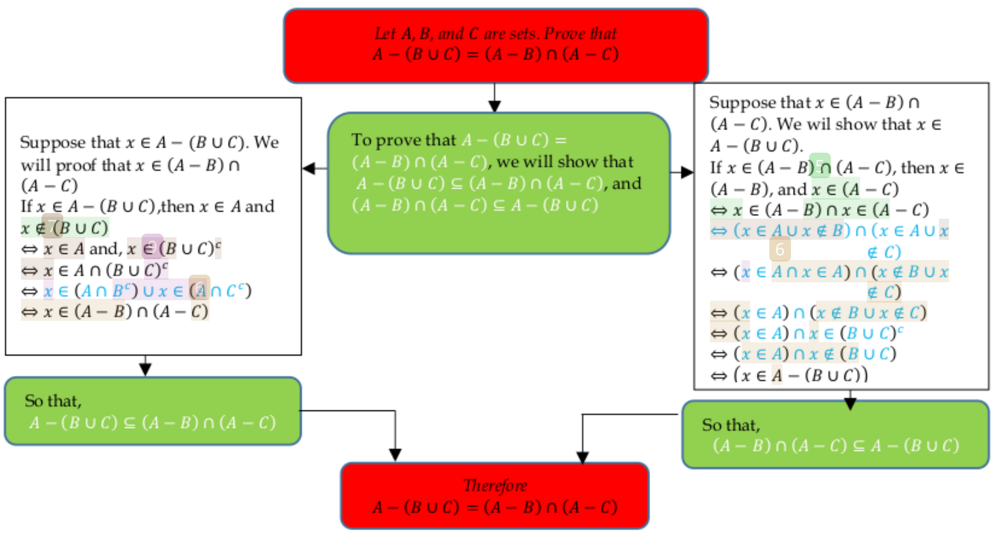

Figure 7. Scheme of unrelated from BSH subject

Based on Figure 7 above. An error occurred in showing the proof that $A-(B\cup C)\subseteq(A-B)\cap(A-C)$, an error occurred in writing the argument $x\in A\cap B^c$or $x\in A\cap C^c$ because there is no connected between $x\in A\cap(B\cup C)^c$ with $x\in(A\cap B^c)\cup x\in(A\cap C^c)$, and $x\in(A\cap B^c)\cup x\in(A\cap C^c)$ with $x\in(A-B)\cap(A-C)$. This is because there is no $(B\cup C)^c=B^c\cap C^c$ scheme. While in the construction of proof $(A-B)\cap(A-C)\subseteq A-(B\cup C)$, the error coccurs again in writing $x\in(A-B)$ and $x\in(A-C)$ then $(x\in A\cup x\notin B)\cap(x\in A\cup x\notin C)$, resulting in $(x\in A\cap x\in A)\cap(x\notin B\cup x\notin C)$, and finally the error occurs at $(x\in A)\cap(x\notin B\cup x\notin C)$. These (second) errors occur because the understanding of the difference between the two sets is still low. In contrast to the complete and incomplete schema subject, the schema subject is not concerned with understanding idempotence. So it is necessary to conduct interviews to reflect (accommodation) as well as confirm the unrelated construction proofing scheme. The results of the reflections are transcribed in the following conversation (Table 2).

Tabel 2. Transcript of interview BSH Subject

Researcher/ Subject	**Stimulus or respon**
Researcher	: Based on this proving problem. Suppose A, B, and C are sets. Prove that $A-(B\cup C)=(A-B)\cap(A-C)$. Explain what you understand?
BSH	: Based on definition $A=B \Leftrightarrow A\subseteq B$ and $B\subseteq A$, we will prove that $\Longrightarrow A-(B\cup C)\subseteq(A-B)\cap(A-C)$ $\Longleftarrow (A-B)\cap(A-C)\subseteq A-(B\cup C)$
Researcher	: How do you show that $A-(B\cup C)\subseteq(A-B)\cap(A-C)$?
BSH	: Well, sir. Suppose $x\in A-(B\cup C)$. We will show that $x\in(A-B)\cap(A-C)$
Researcher	: Okay, now elaborate and explain your argument up to $A-(B\cup C)\subseteq(A-B)\cap(A-C)$
BSH	: Yes, sir. If $x\in A-(B\cup C)\Rightarrow x\in A$ and $x\notin(B\cup C)$ $\Leftrightarrow x\in A\cap(B\cup C)^c$ $\Leftrightarrow x\in A\cap B^c\ \cup x\in A\cap C^c$ $\Leftrightarrow x\in(A-B)\cap(A-C)$ So that $A-(B\cup C)\subseteq(A-B)\cap(A-C)$
Researcher	: Okay, now you write down the complement definition of a set
BSH	: Eeee, may I have a look at the book, sir!
Researcher	: Suppose $S=\{1,2,3,4\}$ dan $A=\{3,4\}$, then the element S which is not element A is ...
BSH	: $A^c=\{1,2\}$
Researcher	: So, if $x\notin(B\cup C)$, then ...
BSH	: $x\in A$ and $x\in(B\cup C)^c$
Researcher	: Now write down a complementary definition of a set of the known universe.
BSH	: Let S be the set of universes and $A\subseteq S$. Then $A^c=\{x\mid x\in S$ and $x\notin A\}$
Researcher	: Now rewrite the theorem (de morgan's law), to continue your proof.
BSH	: If A and B are sets, then $(A\cup B)^c=A^c\cap B^c$. So, if $x\in A$ and $x\in(B\cup C)^c$, then $\Leftrightarrow x\in A$ and $x\in B^c\cap x\in C^c$ $\Leftrightarrow x\in A$ and $x\in B^c\cap x\in A$ and $x\in C^c$ $\Leftrightarrow x\in A$ and $x\notin B\cap x\in A$ and $x\notin C$ $\Leftrightarrow x\in(A-B)\cap(A-C)$ So that $A-(B\cup C)\subseteq(A-B)\cap(A-C)$
Researcher	: Good. Now, please improve the proof construction from $(A-B)\cap(A-C)\subseteq A-(B\cup C)$, with due regard to the previous properties, definitions and theorems.
BSH	: Yes, sir. Suppose $x\in(A-B)\cap(A-C)$. It will be shown that $x\in A-(B\cup C)$ If $x\in(A-B)\cap(A-C)\Rightarrow x\in(A-B)$ and $x\in(A-C)$ $\Leftrightarrow\ (x\in A$ and $x\notin B)$ and $(x\in A$ and $x\notin C)$ $\Leftrightarrow\ (x\in A$ and $x\in A)$ and $(x\notin B$ and $x\notin C)$ $\Leftrightarrow\ x\in A$ and $(x\in B^c$ and $x\in C^c)$ $\Leftrightarrow\ x\in A$ and $x\in B^c\cap C^c$ $\Leftrightarrow\ x\in A$ and $x\in(B\cup C)^c$ $\Leftrightarrow\ x\in A$ and $x\notin(B\cup C)$ $\Leftrightarrow\ x\in A-(B\cup C)$ So that $(A-B)\cap(A-C)\subseteq A-(B\cup C)$
Researcher	: Right. Now match your current answer with your previous answer.
BSH	: It turns out that the definition of difference, slice, and de morgan's law are the keys to this proof, sir.

After reflection, BSH realizes that understanding the previous axioms, definitions and theorems greatly influences the ability to construct proof. Based on the results of the analysis of the answer sheets and interview results as well as documentation from the subject of proof are not related, theoretically when the BSH subject will construct proof, the new structure of proof (understanding) is not in accordance with the existing schema structure, resulting in dis-equilibrium (imbalance) in the mind that causes a strong attempt on the subject to change the structure of the proof in the interview activity, so that the structure of the proof that he has just faced can be linked (assimilated), so that then equilibrium occurs (balance).

Scheme of Immature Proof

Subjects with immature proof schemes are subjects with invalid theorem construction results and are not based on an understanding of previous (illogical) axioms, definitions, and theorems. The subjects of the immature proof scheme were 12 students. One of them is LZP (initials). The proof construction of the LZP is as follows.

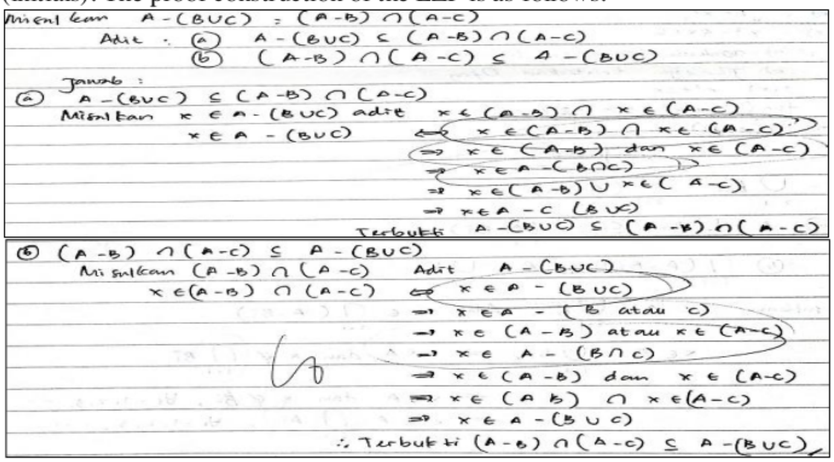

Figure 8. Answer Sheet of LZP subject

Figure 8 above is the answer sheet of the immature evidence subject. If the above answer sheet is redrawn in the form of a proof scheme, then the proof scheme unrelated to LZP can be presented with the following scheme.

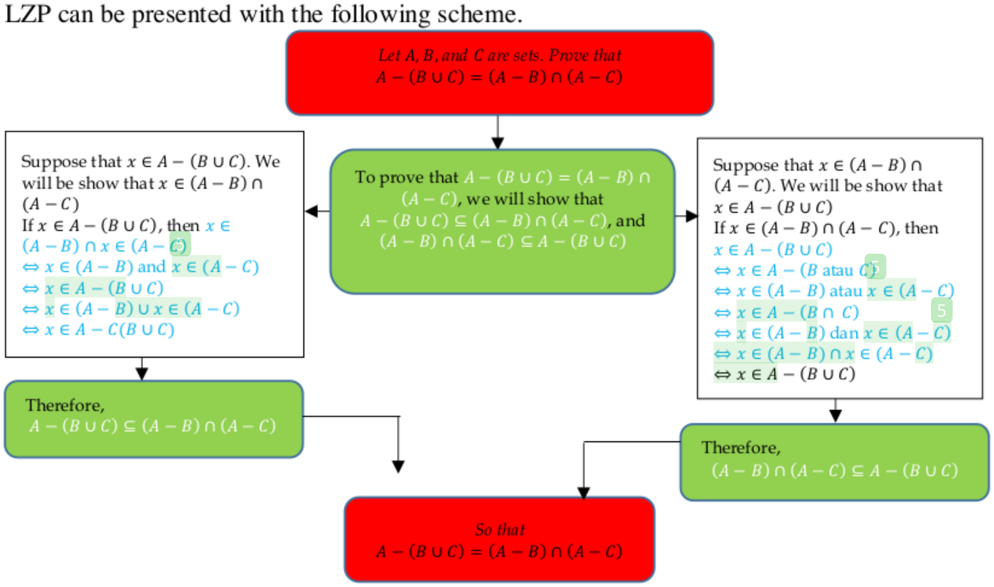

Figure 9. Scheme of Immature Proof from LZW subject

Based on the schematic Figure 9 above, an error occurs at every step of the construction of proof, this is due to 1) weak understanding due to not understanding the integration of real analysis courses with other or previous courses, 2) understanding of logic still has to be improved (if ... then ...), 3) understanding of the axioms, definitions and theorems that have been proven beforehand, so it is not possible to see that the new theorems are built on previous axioms, definitions and theorems. So that these weaknesses need to be corrected and arranged through interviews with the aim of reflection (accommodation) on the understanding of the ability to construct proof based on previous axioms, definitions, and theorems. The results of the reflections are transcribed in the following conversation (Table 3).

Table 3. Transcript of interview LZP Subject

Researcher / Subjek		**Stimulus or respon**
Researcher	:	Based on this proving problem. Suppose A, B, and C are sets. Prove that: $A-(B\cup C)=(A-B)\cap(A-C)$. Explain what your understand?
LZP	:	We will show that $\Rightarrow A-(B\cup C)\subseteq(A-B)\cap(A-C)$ $\Leftarrow (A-B)\cap(A-C)\subseteq A-(B\cup C)$
Researcher	:	What underlies you want to show: $\Rightarrow A-(B\cup C)\subseteq(A-B)\cap(A-C)$ $\Leftarrow (A-B)\cap(A-C)\subseteq A-(B\cup C)$ To proof that $A-(B\cup C)=(A-B)\cap(A-C)$
LZP	:	Where did you come from, sir? Is there any basis for us to do proof like this proof problem?
Researcher	:	Try to understand every definition in the books and teaching materials that we use in real analysis courses.
LZP	:	Yes, sir. Definition "Suppose $A=B$ if and only if $A\subseteq B$ and $B\subseteq A$"
Researcher	:	Good. Now, how do you show that $-(B\cup C)\subseteq(A-B)\cap(A-C)$?
LZP	:	Well, sir. Suppose $x\in A-(B\cup C)$. We will show that $x\in(A-B)\cap(A-C)$
Researcher	:	Okay, now lay out your argument.
LZP	:	If $x\in A-(B\cup C)$, then $(A-B)\cap(A-C)$
Researcher	:	Please waiting for a minute, try to write down the definition of the difference between two sets.
LZP	:	Will, sir. "$A-B=\{x \mid x\in A$ dan $x\notin B\}$"
Researcher	:	Right, So if $x\in A-(B\cup C)$ then ...
LZP	:	While writing, if $x\in A-(B\cup C)$ then $x\in A$ and $x\notin(B\cup C)$
Researcher	:	Now continue
LZP	:	If $x\in A-(B\cup C)$, then $x\in A$ and $x\notin(B\cup C)$ While opening the sheets of books and teaching materials and asking whether this argument has anything to do with the definition of complement set sir?
Researcher	:	Good,
LZP	:	So, if $x\in A$ dan $x\notin(B\cup C)$, then $x\in A$ and $x\in(B\cup C)^c$
Peneliti	:	Before continuing with this evidence construction process (pointing), what did you catch from each of the definitions used?
LZP	:	Definitions can support every step of the evidence construction argument we pack.
Researcher	:	Well now get back to continue the proof construction done.
LZP	:	If $x\in A$ and $x\notin(B\cup C)$, then $x\in A$ and $x\in(B\cup C)^c$ $\Leftrightarrow x\in A$ and $x\in B^c\cap C^c$ While opening the book and teaching materials (the complement definition "Suppose S is universal set and $A\subseteq S$. Then $A^c=\{x \mid x\in S$ and $x\notin A\}$"), and get back to writing: $\Leftrightarrow x\in A$ and $(x\in B^c$ and $x\in C^c)$ $\Leftrightarrow (x\in A$ and $x\in B^c)$ and $(x\in A$ and $x\in C^c)$ $\Leftrightarrow (x\in A$ and $x\notin B)$ and $(x\in A$ and $x\notin C)$ $\Leftrightarrow x\in(A-B)$ dan $x\in(A-C)$

Researcher / Subjek		Stimulus or respon
		$\Leftrightarrow\ x \in (A - B) \cap (A - C)$ So that $A - (B \cup C) \subseteq (A - B) \cap (A - C)$
Researcher	:	Good. Try it now, improve the proof construction from $(A - B) \cap (A - C) \subseteq A - (B \cup C)$, with due regard to the previous properties, definitions and theorems.
LZP	:	Yes, sir. But may I go back to looking at the books, to ensure the correctness of my evidence construction (it seems that the LZP lacks confidence in the understanding it just got).
Researcher	:	Good. But dependence (book stimulus) must be reduced.
LZP	:	Yes, sir. So this is it, sir Suppose that $x \in (A - B) \cap (A - C)$. We will be shown that $x \in A - (B \cup C)$ If $x \in (A - B) \cap (A - C) \Rightarrow x \in (A - B)$ and $x \in (A - C)$ $\Leftrightarrow\ (x \in A$ and $x \notin B)$ and $(x \in A$ and $x \notin C)$ $\Leftrightarrow\ (x \in A$ and $x \in A)$ and $(x \notin B$ and $x \notin C)$ $\Leftrightarrow\ x \in A$ and $(x \in B^c$ and $x \in C^c)$ $\Leftrightarrow\ x \in A$ and $x \in B^c \cap C^c$ $\Leftrightarrow\ x \in A$ and $x \in (B \cup C)^c$ $\Leftrightarrow\ x \in A$ and $x \notin (B \cup C)$ $\Leftrightarrow\ x \in A - (B \cup C)$ So that $(A - B) \cap (A - C) \subseteq A - (B \cup C)$
Researcher	:	Good. With the help of books, your proof is correct. Now how do you feel after being able to correct your mistakes?
LZP	:	Thank God, happy sir. Understanding of the previous axioms, definitions, de Morgan's laws, and theorems is very helpful when trying to construct a new theorem.

Based on the results of the analysis of the answer sheets that are categorized into immature schemes, and the results of interviews or reflections on the answer sheets given. Finally, the LZP subject realizes that understanding the previous axioms, definitions, properties, and theorems greatly affects the ability to construct proof. Realizing here is more about the ability to see that the previous definitions, properties and theorems are part of a unified whole.

According to Piaget, if someone wants to construct new knowledge/information, it means that he wants to link the new information into the schema in his mind, and has two possibilities, namely (1) first, if the new information structure is in accordance with the existing structure in the scheme. So that the information can be linked into and integrated into the scheme, a construction process called assimilation occurs, and (2) second, if the new information structure does not match the schematic structure, there will be a dis-equilibrium (imbalance) in the mind which causes a strong urge in the person to change the structure of the schema so that the new information can be linked (assimilated), then equilibrium occurs again, so this second process is called accommodation (Sutawidjaja & Afgani, 2015;Netti et al., 2017; Subanji, 2006; Subanji & Supratman, 2015). Between structure and schema, according to Subanji (2006) our cognitive structure is a schemata, which is a collection of schemas (structures). Individuals can remember, understand, and respond to stimuli due to the working of these schemes.

The results of the analysis of the answer sheets and interviews based on the responses of the student answer sheets, it can be seen that what causes the lack of ability to construct student proof is the student's understanding of the structure of the proof itself. The proof structure here is knowledge of previous definitions, properties and theorems. Previous definitions, properties, and theorems can basically be used as materials for constructing or constructing new theorems or mathematical statements. Between understanding the proof and construction of proof, must be coherent. Because it will be a problem if you focus too much on proof construction rather than understanding the proof (Hodds et al., 2014). Furthermore

Hodds et al. (2014) said that understanding arguments which are mathematical evidence can prevent students from giving examples and choosing to give deductive arguments.

Understanding concepts and understanding arguments (mathematical proof) are not the same thing. Understanding mathematical concepts can be characterized by providing/distinguishing which ones are examples or not. Meanwhile, understanding proof is giving arguments from one or more premises to a conclusion that can convince others. Increasing understanding of the evidence for most teachers is not easy, but that doesn't mean it can't (Hodds et al., (2014). The ways to increase understanding of proof according to are a) changing the presentation of proof and (b) changing the way students engage with proof; and self-explanation training, and generic proof from (Lew et al., 2020).

With the understanding possessed by students such as an understanding of the definitions, properties, and theorems that have been proven before, it can be new material to build or construct a proof of new mathematical theorems or statements. Thus in constructing proof students can link several other proof structures and develop a more mature proof scheme. This is in accordance with the opinion Rav (1999) which says that the need to understand especially writing proof is very important, because studying proof is learning new ideas, new concepts, new strategies that can be assimilated into further development research. The entire arsenal of mathematical methodologies, concepts, strategies and techniques for solving problems, forming interconnections, and all mathematical knowledge is embedded in proof.

CONCLUSION

The ability to prove can be divided into two, namely the ability to construct and validate proof. Then the student's ability to prove a given mathematical statement is explored through 4 (four) forms of proof schemes, namely 1) a complete proof scheme consisting of 2 subjects; 2) the proof scheme is incomplete, consisting of 13 subjects; 3) the proof scheme is unrelated, consisting of 8 subjects; and 4) immature proof scheme, consisting of 12 subjects. Exploration of students' proving abilities is influenced by: 1) understanding and the ability to see that new theorems are built on previous definitions, properties and theorems, and 2) how to present a proof and how students engage with a proof. Suggestions in this study are 1) changing the way of presenting proof, and changing how students are involved in a proof, 2) increasing understanding through proving routine mathematical theorems or statements, and 3) developing lecture designs that can turn proving activities into activities routine, not non-routine.

RECOMMENDATION

First, this research is still limited to simple mathematical problems / statements, so it is necessary to study more complex problems / statements in order to find different schemes of proof. Second, it is necessary to study how to improve student understanding based on developing theories.

ACKNOWLEDGEMENT

The author thanks Drs. Ketut Sarjana, M.S as a tutor and partner in the Real Analysis course in the Mathematics Education study program, FKIP University of Mataram, who has guided and provided opportunities for writers to guide and participate in activities during the lecture.

REFERENCES

Alcock, L., & Weber, K. (2005). Proof validation in real analysis: Inferring and checking warrants. *Journal of Mathematical Behavior*. https://doi.org/10.1016/j.jmathb.2005.03.003

Anwar, L., Nasution, S. H., Sudirman, & Susiswo. (2018). Proses Berpikir Mahasiswa Dalam

Membuktikan Proposisi: Konseptualisasi-Gambar. *Jurnal Kajian Pembelajaran Matematika (JKPM)*, *2*(2), 46–56.

Bartle, R. G., & Sherbert, D. R. (2011). *Introduction to Real Analysis* (Fourth Edi). John Wiley & Sons, Inc.

Botts, T., & Royden, H. L. (1964). Real Analysis. In *The American Mathematical Monthly* (Vol. 71, Issue 9). https://doi.org/10.2307/2311951

Devlin, K. (2003). Sets, Functions, and Logic: An Introduction to Abstract Mathematics. In *Sets, Functions, and Logic* (Third Edit). CRC Press.

Hamdani, D., Junaidi, J., Novitasari, D., Salsabila, N. H., & Tyaningsih, R. Y. (2020). Bukti yang Membuktikan dan Bukti yang Menjelaskan dalam Kelas Matematika. *Jurnal Penelitian Dan Pengkajian Ilmu Pendidikan: E-Saintika*, *4*(2), 248. https://doi.org/10.36312/e-saintika.v4i2.253

Harini, L., Astawa, G., & Srinadi, G. (2014). Eksplorasi Miskonsepsi Mahasiswa Dalam Pengembangan Buku Teks Analisis Real Bermuatan Peta Pikiran. *Seminar Nasional Sains Dan Teknologi*, *September*, 941–949. https://www.researchgate.net/publication/285592489_EKSPLORASI_MISKONSEPSI_MAHASISWA_DALAM_PENGEMBANGAN_BUKU_TEKS_ANALISIS_REAL_BERMUATAN_PETA_PIKIRAN

Hernadi, J. (2013). Metoda Pembuktian dalam Matematika. *Jurnal Pendidikan Matematika*, *2*(1), 1–13. https://doi.org/10.22342/jpm.2.1.295.

Hodds, M., Alcock, L., & Inglis Loughborough, M. (2014). Self-explanation training improves proof comprehension. *Journal for Research in Mathematics Education*, *45*(1), 62–101. https://doi.org/10.5951/jresematheduc.45.1.0062

Juandi, D. (2008). Pembuktian, Penalaran, Dan Komunikasi Matematik. *JurDikMat FPMIPA UPI*.

Lew, K., Weber, K., & Mejía Ramos, J. P. (2020). Do Generic Proofs Improve Proof Comprehension? *Journal of Educational Research in Mathematics*. https://doi.org/10.29275/jerm.2020.08.sp.1.229

Miyazaki, M., Fujita, T., & Jones, K. (2017). Students' understanding of the structure of deductive proof. *Educational Studies in Mathematics*. https://doi.org/10.1007/s10649-016-9720-9

Morash, R. P. (1987). Bridge to abstract mathematics: Mathematical Proof and Structures. In *Choice Reviews Online* (Vol. 50, Issue 06). Random House, Inc. https://doi.org/10.5860/choice.50-3317

National Council of Teachers of Mathematics. (2000). Principles and Standards for School Mathematics. In *School Science and Mathematics*. The Council.

Netti, S., & Herawati, S. (2019). Characteristics of Undergraduate Students' Mathematical Proof Construction on Proving Limit Theorem. *KnE Social Sciences*, *3*(15), 153–163. https://doi.org/10.18502/kss.v3i15.4362

Netti, S., Nusantara, T., Subanji, S., Abadyo, A., & Anwar, L. (2016). The Failure to Construct Proof Based on Assimilation and Accommodation Framework from Piaget. *International Education Studies*, *9*(12), 11. https://doi.org/10.5539/ies.v9n12p12

Netti, S., Sutawidjaja, A., & Mulyati, S. (2017). Skema Berpikir Mahasiswa Ketika Mengostruksi Bukti Matematis. *Prosiding SI MaNIs (Seminar Nasional Integrasi Matematika Dan Nilai Islami)*, *1*(1), 547–555.

Rav, Y. (1999). Why Do We Prove Theorems? *Philosophia Mathematica*, *7*, 5–41. https://doi.org/10.1093/philmat/7.1.5

Selden, A., & Selden, J. (2003). Validations of proofs considered as texts: Can undergraduates tell whether an argument proves a theorem? *Journal for Research in Mathematics Education*, *34*(1), 4–36. https://doi.org/10.2307/30034698

Shadiq, F. (2015). *Pembuktian Tidak Langsung* (pp. 1–6). PPPPTK Matematika. http://p4tkmatematika.kemdikbud.go.id/artikel/2015/02/12/pembuktian-tak-langsung/

Subanji. (2006). Berpikir Pseudo Penalaran Kovariasi dalam Mengkonstruksi Grafik Fungsi Kejadian Dinamik : Sebuah Analisis Berdasarkan Kerangka Kerja VL2P dan Implikasinya pada Pembelajaran Matematika. *Jurnal Ilmu Pendidikan*, *13*(1), 1–8.

Subanji, R., & Supratman, A. M. (2015). The Pseudo-Covariational Reasoning Thought Processes in Constructing Graph Function of Reversible Event Dynamics Based on Assimilation and Accommodation Frameworks. *Research in Mathematical Education*. https://doi.org/10.7468/jksmed.2015.19.1.61

Sutawidjaja, A., & Afgani, J. (2015). Konsep Dasar Pembelajaran Matematika. *Pembelajaran Matematika*.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2012). *Elementary and Middle School Mathematics: Teaching Developmentally* (Seventh Ed). Allyn & Bacon is an Imprint of Pearson.

Weber, K. (2003). Research Sampler 8: students' difficulties with proof. *The Mathematical Association of America: Online*, *1*, 1–8. http://www.maa.org/programs/faculty-and-departments/curriculum-department-guidelines-recommendations/teaching-and-learning/research-sampler-8-students-difficulties-with-proof

Wikipedia contributors. (2020). *Theorem*. Wikipedia, The Free Encyclopedia. https://en.wikipedia.org/w/index.php?title=Theorem&oldid=944519294

14. Ketut Sarjana

ORIGINALITY REPORT

11%	6%	9%	7%
SIMILARITY INDEX	INTERNET SOURCES	PUBLICATIONS	STUDENT PAPERS

PRIMARY SOURCES

1	www.numerade.com Internet Source	2%
2	Submitted to Navitas Global Student Paper	2%
3	G Emeira, Hapizah, Scristia. "Mathematical proof analysis using mathematical induction of grade XI students", Journal of Physics: Conference Series, 2020 Publication	2%
4	ojs.ikipmataram.ac.id Internet Source	1%
5	Submitted to The University of Manchester Student Paper	1%
6	Xihe, L.. "Stability of random membership frequency and fuzzy statistics", Fuzzy Sets and Systems, 19890110 Publication	1%
7	Submitted to University of Bristol Student Paper	1%

8	Dean Corbae, Sam Ouliaris, Peter C. B. Phillips. "Band Spectral Regression with Trending Data", Econometrica, 2002 Publication	<1 %
9	Holl, A.. "Covariant linear response theory of relativistic QED plasmas", Physica A: Statistical Mechanics and its Applications, 20030301 Publication	<1 %
10	Keith Weber. "Effective Proof Reading Strategies for Comprehending Mathematical Proofs", International Journal of Research in Undergraduate Mathematics Education, 2015 Publication	<1 %
11	"Encyclopedia of Mathematics Education", Springer Science and Business Media LLC, 2020 Publication	<1 %
12	"Advances in Mathematics Education Research on Proof and Proving", Springer Science and Business Media LLC, 2018 Publication	<1 %

Exclude quotes	On	Exclude matches	Off
Exclude bibliography	On		

14. Ketut Sarjana

GRADEMARK REPORT

FINAL GRADE

/0

GENERAL COMMENTS

Instructor

PAGE 1

PAGE 2

PAGE 3

PAGE 4

PAGE 5

PAGE 6

PAGE 7

PAGE 8

PAGE 9

PAGE 10

PAGE 11

PAGE 12

PAGE 13

PAGE 14